NTI

Non-Traditional Instruction

Math

Days 11-20

**Work will be modified according to each student's IEP or 504 plan

GRADES WILL BE POSTED DAILY PER MR. BENNETTS REQUEST. PLEASE SUBMIT A PICTURE OF YOUR COMPLETED WORK. IF YOU FINISH THE ENTIRE PACKET OR WORK AHEAD SEND EVERYTHING THAT IS COMPLETE. HERE IS A BREAKDOWN OF HOW MANY PROBLEMS YOU SHOULD COMPLETE EACH DAY.

March 16 DAY 6 #1-10
March 17 DAY 7 #11-20
March 18 DAY 8 #21-30
March 19 DAY 9 #31-40
March 23 DAY 10 #41-50
March 25 DAY 11 #51-60
March 26 DAY 12 #61-70
March 27 DAY 13 #71-80
March 30 DAY 14 #81-90
April 1 DAY 15 #91-100
April 2 DAY 16 #101-105
April 3 DAY 17 #106-110

If you don't have internet access you may turn in work to the food truck. Please make sure it has your name and Carter at the top of each page that you are turning in. You may turn it in every Monday to the food truck or school. I know who the ones are without access. Everyone else is required to turn in daily pictures.

Algebra 1 Name________________________________ ID: 1

NTI DAYS 6-17

Date________________ Period____

Solve each equation.

1) $-7n = -119$

2) $-19 = \frac{p}{13}$

3) $-2 = a + 5$

4) $-4 = a - 8$

5) $288 = 18k$

6) $8 = \frac{k + 2}{2}$

7) $-27 = 2x + 5$

8) $-1 = \frac{k - 10}{8}$

9) $3p - 1 = 5$

10) $-45 = 5(-6 + n)$

11) $90 = -3p - 8(p - 3)$

12) $-330 = -2 + 8(7r - 6)$

13) $216 = 8(-4v + 3)$

14) $234 = -6(7r - 4)$

15) $147 = 7v - 7(-5 + 3v)$

16) $|a + 8| = 11$

17) $\left|\frac{x}{2}\right| = 5$

18) $\left|\frac{x}{7}\right| = 3$

19) $|-10 + a| = 9$

20) $|p - 2| = 7$

Draw a graph for each inequality.

21) $p \geq -4$

−7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7

22) $n < 2$

−7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7

23) $a < -6$

−7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7

24) $v > 6$

−7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7

25) $n \geq 2$

−7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7

Solve each inequality and graph its solution.

26) $-7 < n - 17$

3 4 5 6 7 8 9 10 11 12 13

27) $-180 > 18x$

−18 −16 −14 −12 −10 −8

28) $-4 \leq n - 15$

3 4 5 6 7 8 9 10 11 12 13

29) $7 \geq \frac{x}{8}$

48 49 50 51 52 53 54 55 56 57 58

30) $-24 > x - 7$

−24 −22 −20 −18 −16 −14

31) $-30 \geq 5(-4 + a)$

−7 −6 −5 −4 −3 −2 −1 0 1 2 3

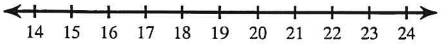
32) $-6a - 2 > -44$

4 5 6 7 8 9 10 11 12 13 14

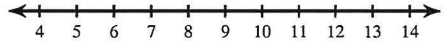
33) $-9(2 + n) > -189$

14 15 16 17 18 19 20 21 22 23 24

34) $-9 - 7n \le -149$

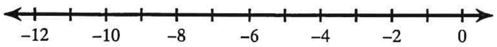


35) $-24 > -2(x + 5)$

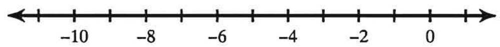


Solve each compound inequality and graph its solution.

36) $-32 \le 4m \le -28$

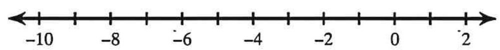


37) $-2 < \frac{r}{4} \le 0$

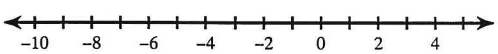


38) $\frac{x}{8} \ge -1$ and $x + 3 < -2$

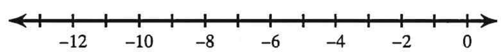
39) $-5r \le 30$ and $r + 6 \le 4$



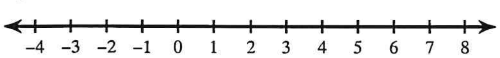
40) $\frac{m}{8} \ge -1$ and $m - 9 < -5$



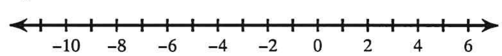
41) $-7x \ge 63$ or $10 + x > 7$



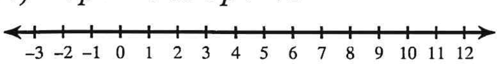
42) $n + 2 \ge 7$ or $n - 7 < -6$



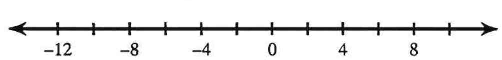
43) $-4k \le -4$ or $-6k > 48$



44) $-7p > -7$ or $9p > 72$

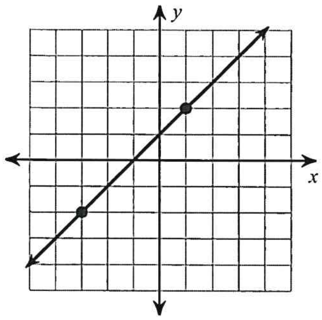


45) $\frac{x}{8} < -1$ or $\frac{x}{7} > 1$

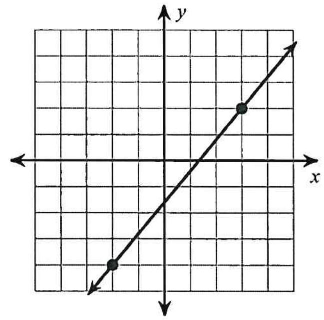


Find the slope of each line.

46)



47)



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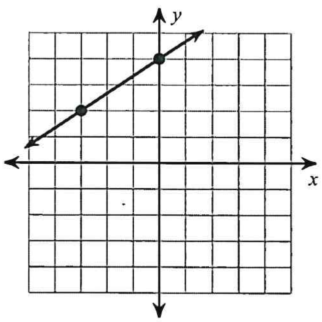

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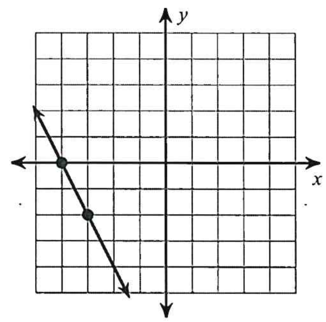

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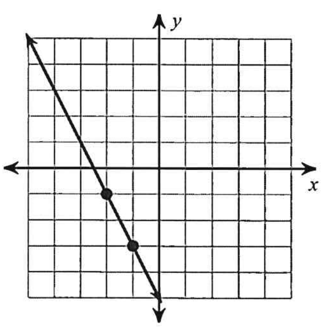


Find the slope of the line through each pair of points.

51) $(2, -20), (-6, -7)$

52) $(2, 14), (-6, 18)$

53) $(-18, -10), (-13, 10)$

54) $(14, 2), (-2, -18)$

55) $(19, -12), (8, -10)$

Find the slope of each line.

56) $y = -1$

57) $y = \frac{3}{5}x - 3$

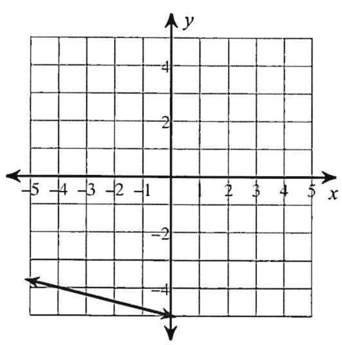
58) $y = -\frac{1}{2}x + 3$

59) $y = -\frac{9}{2}x - 5$

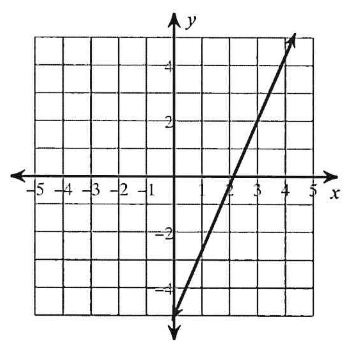
60) $y = -3x - 5$

Write the slope-intercept form of the equation of each line.

61)



62)



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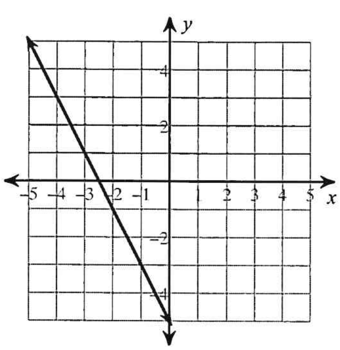

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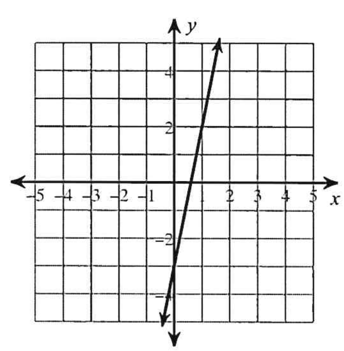
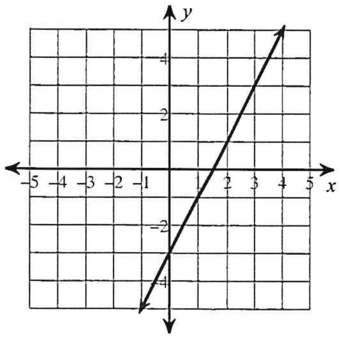


65)



Write the slope-intercept form of the equation of each line given the slope and y-intercept.

66) Slope = 6, y-intercept = 5

67) Slope = −2, y-intercept = −1

68) Slope $= -\frac{3}{4}$, y-intercept = −1

69) Slope = 10, y-intercept = 5

70) Slope $= \frac{5}{3}$, y-intercept = 0

Write the slope-intercept form of the equation of the line through the given point with the given slope.

71) through: $(-5, 1)$, slope $= \frac{3}{5}$

72) through: $(5, 4)$, slope $= -\frac{1}{5}$

73) through: $(3, 1)$, slope = −1

74) through: $(5, 3)$, slope = 4

75) through: $(2, 1)$, slope = −1

Write the slope-intercept form of the equation of the line described.

76) through: $(-5, 0)$, parallel to $y = \frac{4}{5}x + 5$

77) through: $(-5, 2)$, parallel to $y = -\frac{3}{5}x - 2$

78) through: $(4, -1)$, parallel to $y = -\frac{1}{2}x + 2$

79) through: $(-1, 1)$, parallel to $y = 4x + 1$

80) through: $(-2, 2)$, parallel to $y = x + 2$

81) through: $(-3, -1)$, perp. to $y = \frac{3}{4}x + 5$

82) through: $(2, 3)$, perp. to $y = -\frac{2}{3}x - 3$

83) through: $(1, -4)$, perp. to $y = \frac{1}{2}x - 1$

84) through: $(-5, -3)$, perp. to $y = -\frac{9}{2}x$

85) through: $(5, 0)$, perp. to $y = 5x - 5$

86) through: $(-3, -5)$, perp. to $y = -\frac{3}{5}x + 1$

87) through: $(-1, 0)$, perp. to $y = \frac{1}{5}x + 5$

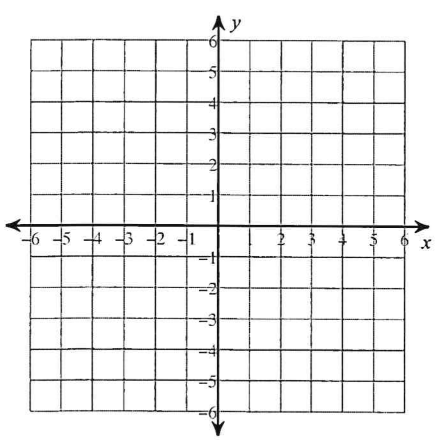
88) through: $(3, 1)$, perp. to $y = -\frac{3}{2}x$

89) through: $(-1, 1)$, perp. to $y = \frac{1}{2}x - 2$

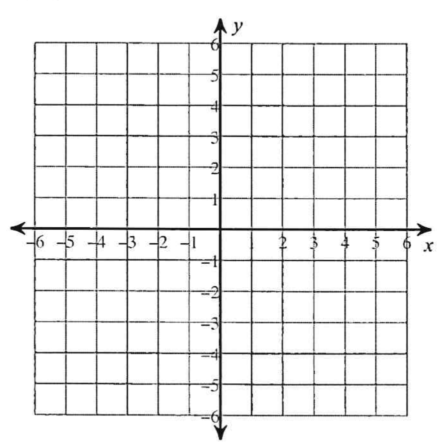
90) through: $(-1, 4)$, perp. to $y = x - 3$

Sketch the graph of each line.

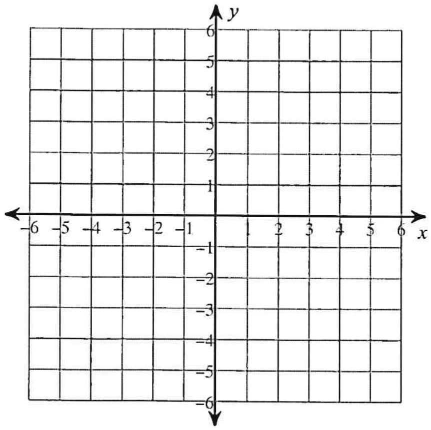
91) $y = -3x - 4$



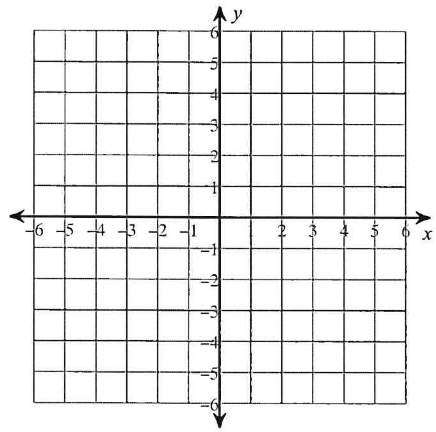
92) $y = -x - 1$



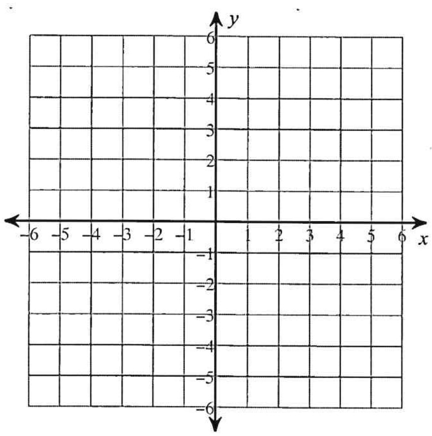
93) $y = -\frac{2}{5}x + 1$



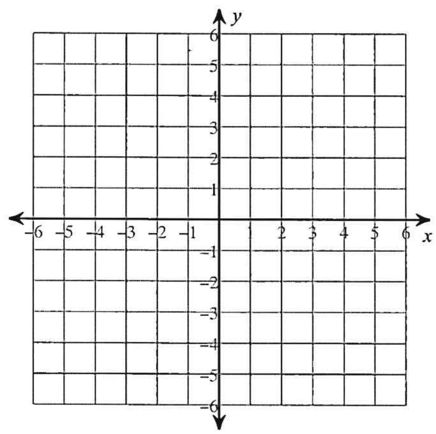
94) $y = -2x - 2$



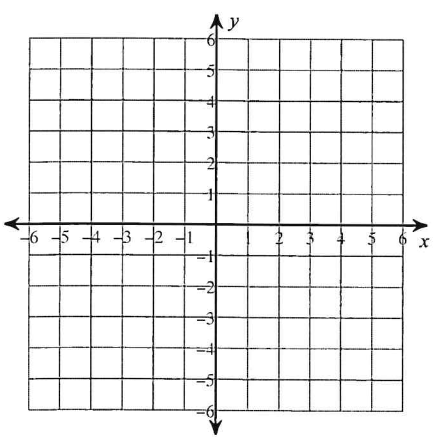
95) $y = -\frac{1}{4}x - 4$



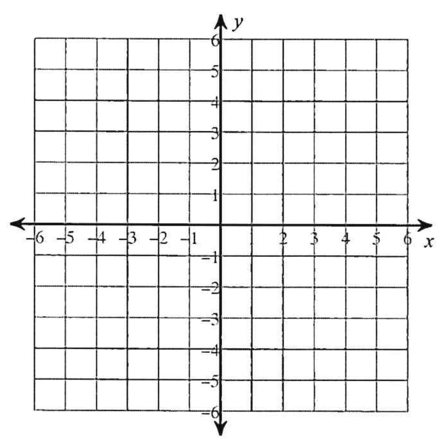
96) $y = -2x + 3$



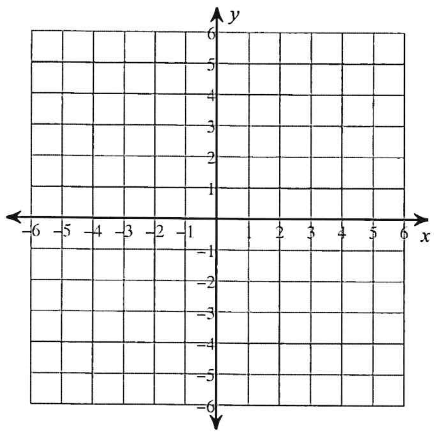
97) $y = \frac{1}{4}x + 2$



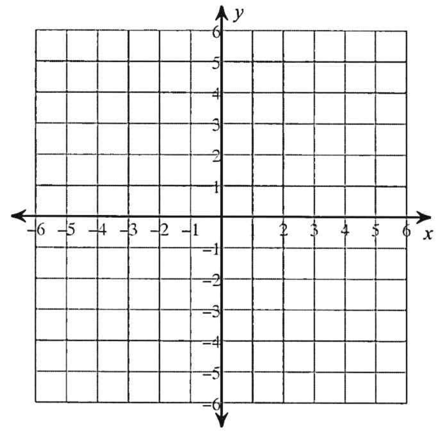
98) $x = -3$



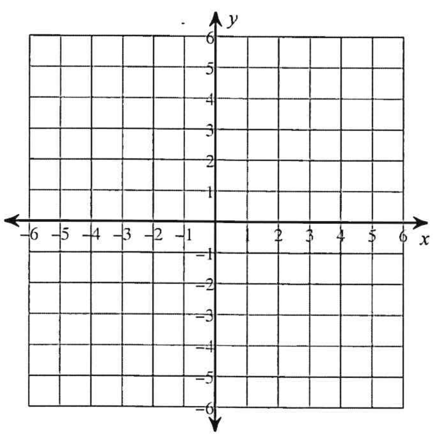
99) $y = \frac{1}{2}x$



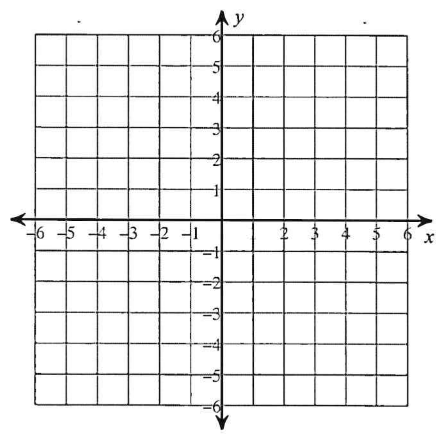
100) $y = -\frac{5}{2}x$



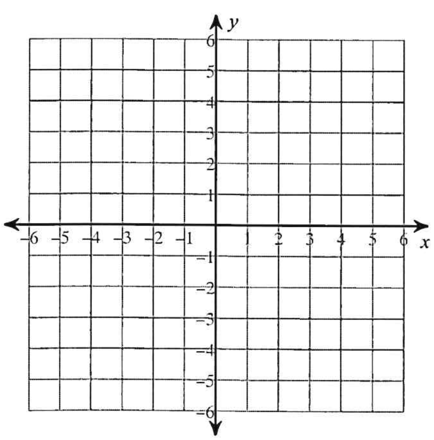
101) $x - y = -2$



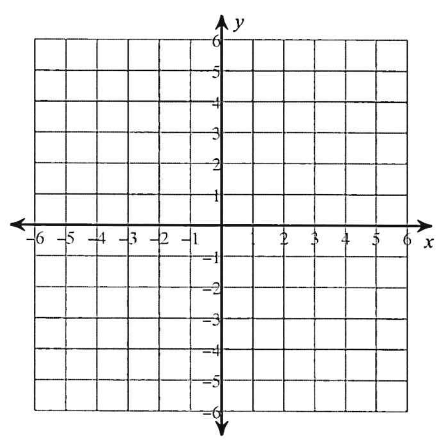
102) $2x + y = 0$



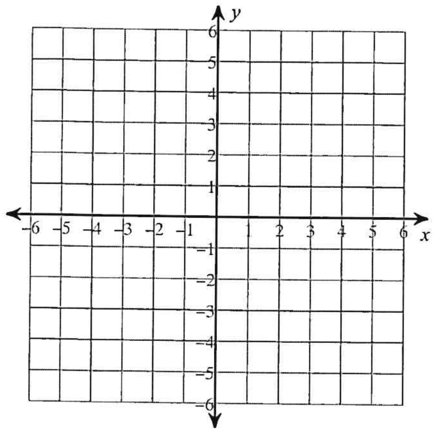
103) $5x + y = -5$



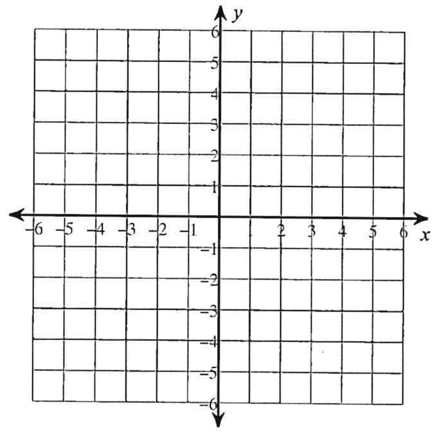
104) $x - y = 3$



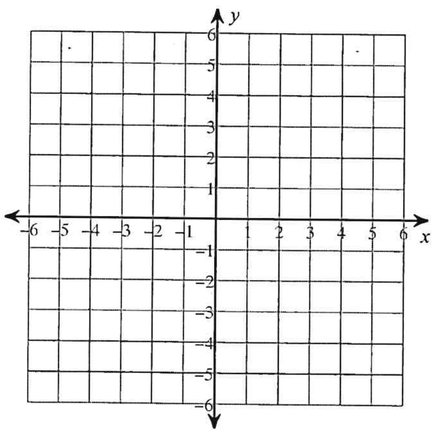
105) $3x - y = -5$



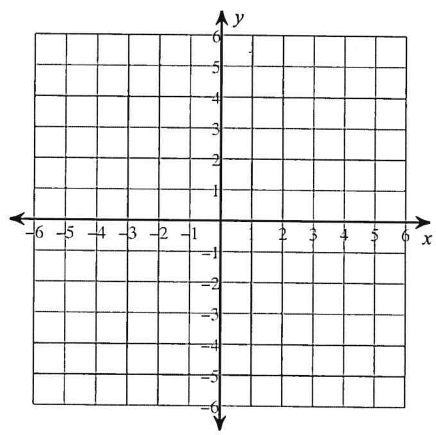
106) $4x + 5y = 0$



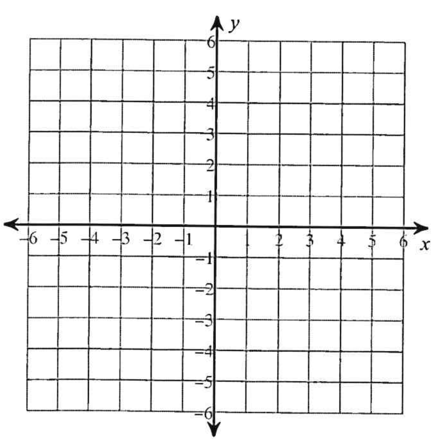
107) $7x + 2y = -4$



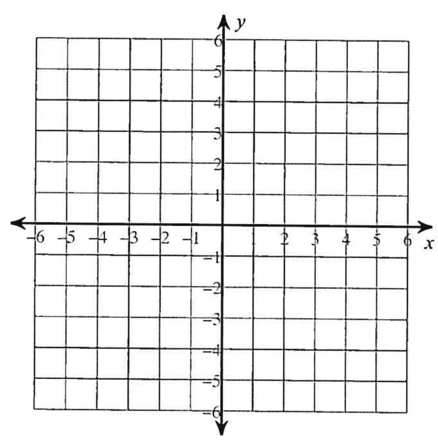
108) $3x - 2y = 6$



109) $3x + 2y = 4$



110) $4x + 3y = -3$



Algebra 1

Name____________________________ ID: 1

NTI DAYS 18-20

Date________________ Period____

NTI DAY 18 Solve each equation.

1) $-30 = 6 + 4n$

2) $-6 + 5b = -86$

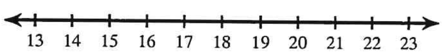
3) $-7(b + 7) = 42$

4) $6(5 + 5n) = -210$

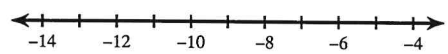
5) $-208 = 8(-3r - 4) - 8$

NTI DAY 19 #6-10 Solve each inequality and graph its solution.

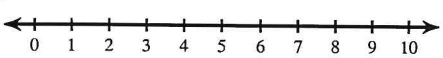
6) $5 + x > 22$



7) $-2 + n < -10$

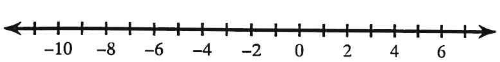


8) $20m \leq 120$

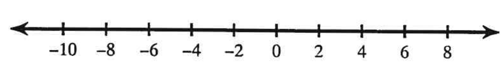


Solve each compound inequality and graph its solution.

9) $\frac{v}{2} < 3$ and $v + 9 \geq 1$

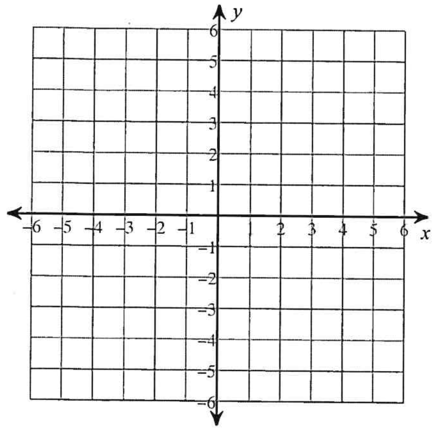


10) $\frac{r}{2} \geq 2$ or $-5r > 30$

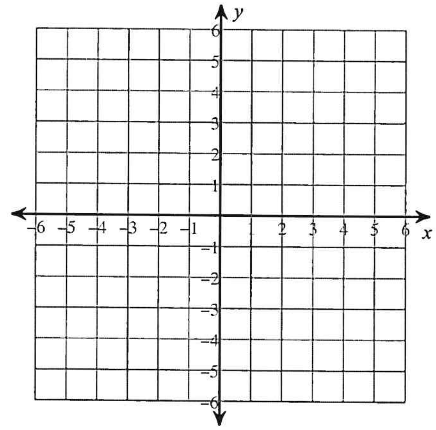


NTI DAY 20 Sketch the graph of each line.

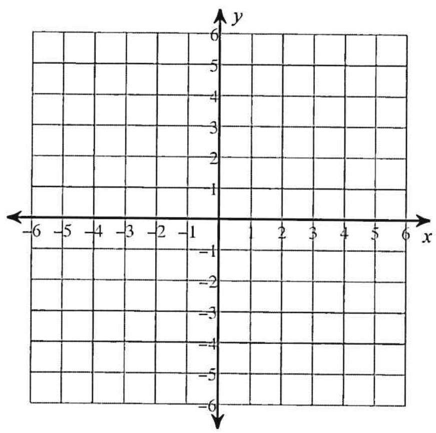
11) $y = -\frac{1}{2}x + 1$



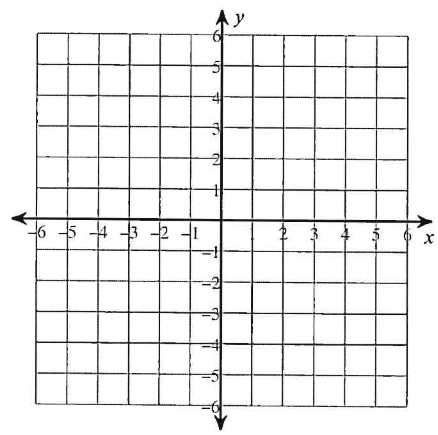
12) $y = \frac{3}{5}x + 5$



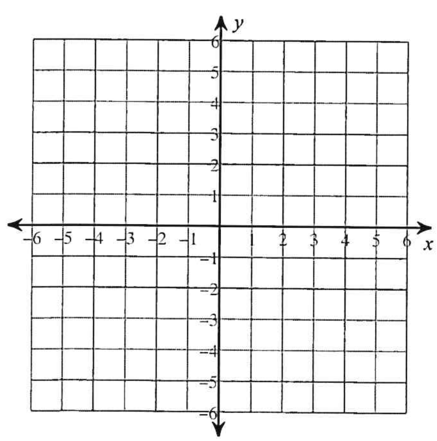
13) $x = -3$



14) $5x - 3y = -9$



15) $3x + y = 3$



Spangler
Geometry

Name: Notes For NTI 9-16

Date:

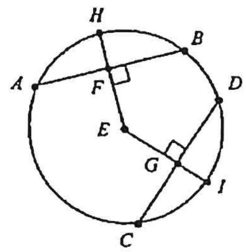
Topic:

Class:

Main Ideas/Questions	Notes/Examples
Congruent **CHORDS & ARCS**	• Two chords are congruent if and only if:

Congruent

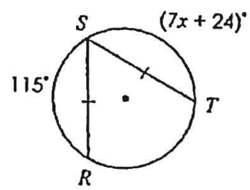
CHORDS & ARCS



- Two chords are congruent if and only if:
  - a) Their corresponding arcs are congruent.
    $AB = CD \leftrightarrow m\widehat{AB} = m\widehat{CD}$
  - b) They are equidistant from the center.
    $AB = CD \leftrightarrow EF = EG$
- If a diameter or radius is perpendicular to a chord, then it bisects the chord and its arc.
  $\overline{EH} \perp \overline{AB} \rightarrow AF = FB$ and $m\widehat{AH} = m\widehat{HB}$

Directions: Find the indicated value.

1. Find x.

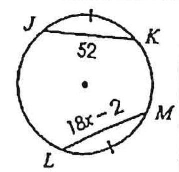


$7x + 24 = 115$

$7x = 91$

$\boxed{x = 13}$

2. Find x.

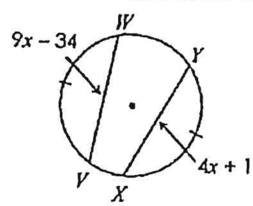


$18x - 2 = 52$

$18x = 54$

$\boxed{x = 3}$

3. Find XY.



$9x - 34 = 4x + 1$

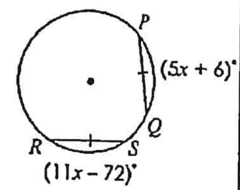
$5x - 34 = 1$

$5x = 35$

$x = 7$

$XY = 4(7) + 1 = \boxed{29}$

4. Find $m\widehat{RS}$.



$11x - 72 = 5x + 6$

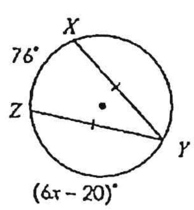
$6x - 72 = 6$

$6x = 78$

$x = 13$

$m\widehat{RS} = 11(13) - 72 = \boxed{71°}$

5. Find x.



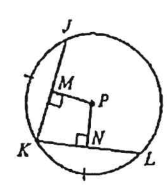
$2(6x - 20) = 284$

$12x - 40 = 284$

$12x = 324$

$\boxed{x = 27}$

6. If $MP = 5x - 34$ and $PN = 2x - 4$, find MP.



$5x - 34 = 2x - 4$

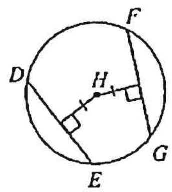
$3x - 34 = -4$

$3x = 30$

$x = 10$

$MP = 5(10) - 34 = \boxed{16}$

7. If $DE = 11x + 15$ and $FG = 32x - 27$, find DE.



$32x - 27 = 11x + 15$

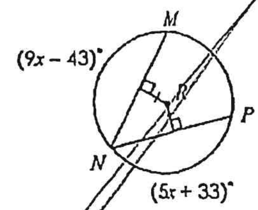
$21x - 27 = 15$

$21x = 42$

$x = 2$

$DE = 11(2) + 15 = \boxed{37}$

8. Find $m\widehat{MP}$.



$9x - 43 = 5x + 33$

$4x - 43 = 33$

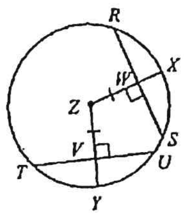
$4x = 76$

$x = 19$

$m\widehat{MN} = 9(19) - 43 = 128°$

$\boxed{m\widehat{MP} = 104°}$

9. In circle Z, if $RS = 18$, and $m\widehat{TY} = 42°$, find each measure.



$TV =$ 9

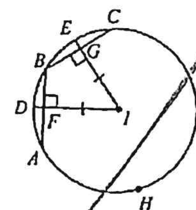
$TU =$ 18

$WS =$ 9

$m\widehat{YU} =$ 42°

$m\widehat{RS} =$ 84°

10. In circle I, if $BG = 17$, and $m\widehat{CHA} = 256°$, find each measure.



$BC =$ 34

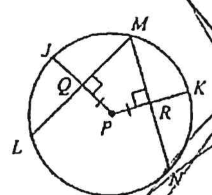
$FB =$ 17

$m\widehat{AB} =$ 52°

$m\widehat{BC} =$ 52°

$m\widehat{EC} =$ 26°

11. If $QM = 6x - 11$ and $MR = 2x + 9$, find MN.



$6x - 11 = 2x + 9$

$4x - 11 = 9$

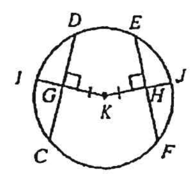
$4x = 20$

$x = 5$

$MR = 2(5) + 9 = 19$

$\boxed{MN = 38}$

12. If $m\widehat{CI} = (7x - 15)°$ and $m\widehat{EF} = (12x - 8)°$, find $m\widehat{CI}$.



$2(7x - 15) = 12x - 8$

$14x - 30 = 12x - 8$

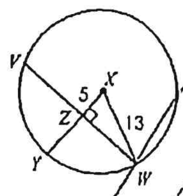
$2x - 30 = -8$

$2x = 22$

$x = 11$

$m\widehat{CI} = 7(11) - 15 = \boxed{62°}$

Use the circle below for questions 13 and 14.



13. Find VW.

$x^2 + 5^2 = 13^2$

$x^2 + 25 = 169$

$x^2 = 144$

$x = 12$

$\boxed{VW = 24}$

14. Find $m\widehat{YW}$.

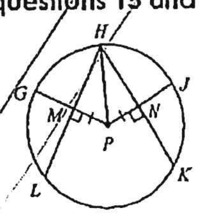
$\cos y = \frac{5}{13}$

$y = \cos^{-1}(5/13)$

$y = 67.4$

$\boxed{m\widehat{YW} = 67.4°}$

Use the circle below for questions 15 and 16.



$HK = 30$ and $PM = 8$

15. Find PH.

$8^2 + 15^2 = x^2$

$289 = x^2$

$17 = x$

$\boxed{PH = 17}$

16. Find $m\widehat{GJ}$.

$\tan y = \frac{15}{8}$

$y = \tan^{-1}\left(\frac{15}{8}\right)$

$y = 61.9$

$\boxed{m\widehat{GJ} = 123.9°}$

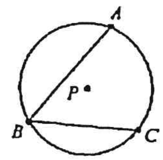
Name:

Date:

Topic:

Class:

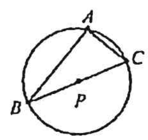
Main Ideas/Questions	Notes/Examples
INSCRIBED ANGLES	• An **inscribed angle** is an angle with its vertex on the circle with two sides that are chords.



$m\angle ABC = \frac{1}{2}\, m\widehat{AC}$

- An **intercepted arc** is the arc that lies between the endpoints of an inscribed angle.
- The **measure of the inscribed angle** is equal to half the measure of its **intercepted arc.**

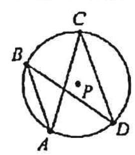
INTERCEPTING a Diameter



If an inscribed angle **intercepts a diameter,** then it is a right angle.

$m\angle BAC =$ 90°

OVERLAPPING Arcs

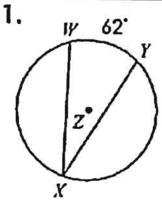


If two inscribed angles intercept the **same arc,** then the angles are congruent.

$m\angle ABD =$ $m\angle ACD$

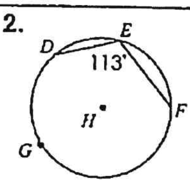
Directions: Find each angle and arc measures.

1.



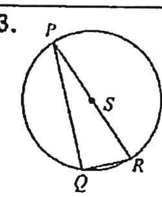
$m\angle WXY =$ 31°

2.



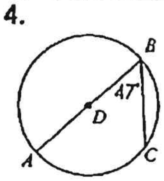
$m\widehat{DGF} =$ 226°

3.



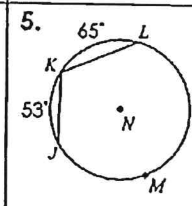
$m\angle PQR =$ 90°

4.



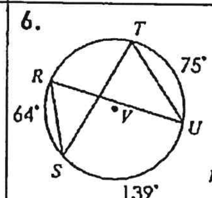
$m\widehat{BC} =$ 86°

5.



$m\angle JKL =$ 121°

6.

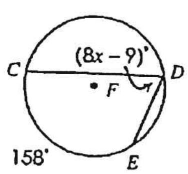


$m\angle RST =$ 41°

$m\angle RUT =$ 41°

Directions: Find each value or measure.

7. Solve for x.

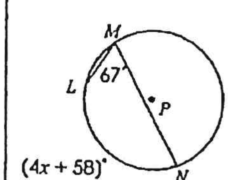


$8x - 9 = 79$

$8x = 88$

$x = 11$

8. Solve for x.



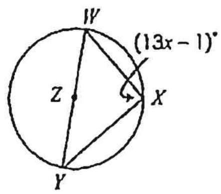
$67 = \frac{1}{2}(4x + 58)$

$67 = 2x + 29$

$38 = 2x$

$x = 19$

9. Solve for x.

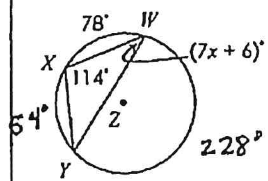


$13x - 1 = 90$

$13x = 91$

$\boxed{x = 7}$

10. Solve for x.

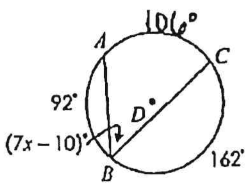


$7x + 6 = 27$

$7x = 21$

$\boxed{x = 3}$

11. Solve for x.

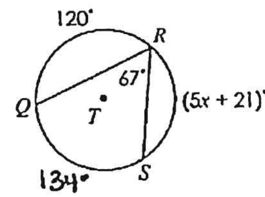


$7x - 10 = 53$

$7x = 63$

$\boxed{x = 9}$

12. Solve for x.

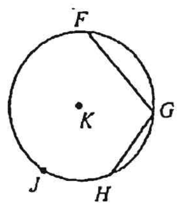


$5x + 21 = 106$

$5x = 85$

$\boxed{x = 17}$

13. If $m\angle FGH = (6x + 21)^\circ$ and $m\overparen{FJH} = (17x - 28)^\circ$, find $m\overparen{FJH}$.



$2(6x + 21) = 17x - 28$

$12x + 42 = 17x - 28$

$42 = 5x - 28$

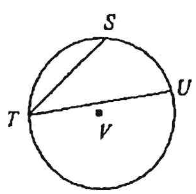
$70 = 5x$

$x = 14$

$m\overparen{FJH} = 17(14) - 28$

$\boxed{= 210^\circ}$

14. If $m\angle STU = (5x - 16)^\circ$ and $m\overparen{SU} = (12x - 50)^\circ$, find $m\angle STU$.



$2(5x - 16) = 12x - 50$

$10x - 32 = 12x - 50$

$-32 = 2x - 50$

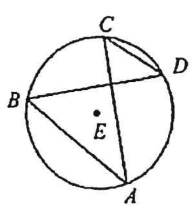
$18 = 2x$

$x = 9$

$m\angle STU = 5(9) - 16$

$\boxed{= 29^\circ}$

15. If $m\angle ABD = (6x + 26)^\circ$ and $m\angle ACD = (13x - 9)^\circ$, find $m\overparen{AD}$.



$6x + 26 = 13x - 9$

$26 = 7x - 9$

$35 = 7x$

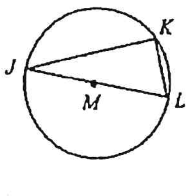
$x = 5$

$m\angle ABD = 6(5) + 26$

$= 56^\circ$

$\boxed{m\overparen{AD} = 112^\circ}$

16. If $m\angle KJL = (3x + 2)^\circ$ and $m\angle KLJ = (7x - 32)^\circ$, find $m\overparen{KL}$.



$10x - 30 = 90$

$10x = 120$

$x = 12$

$m\angle KJL = 3(12) + 2 = 38^\circ$

$\boxed{m\overparen{KL} = 76^\circ}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples

STANDARD FORM EQUATION of a circle

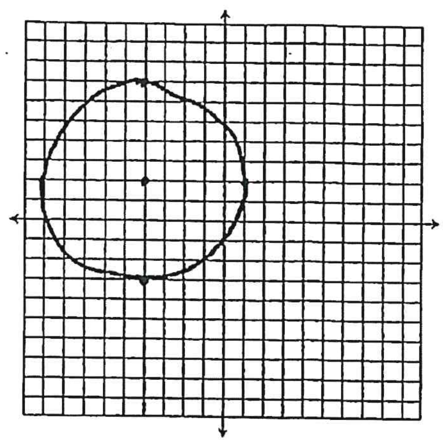
The **standard form** for the equation of a circle in the coordinate plane is given as:

$$(x-h)^2 + (y-k)^2 = r^2$$

where (h,k) is the center and r is the radius.

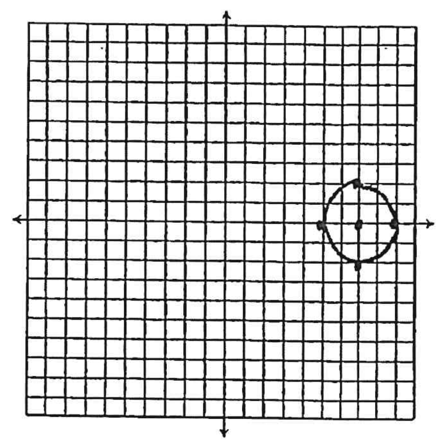
Graph each circle and identify its center and radius.

1. $(x+4)^2+(y-2)^2=25$



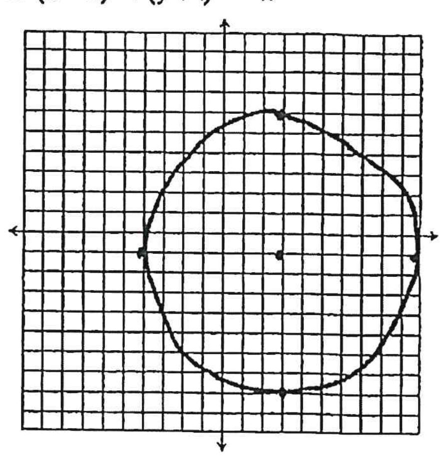
Center:	Radius:
(-4, 2)	5

2. $(x-7)^2+y^2=4$



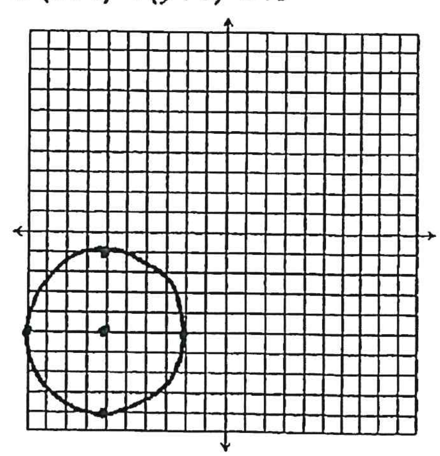
Center:	Radius:
(7, 0)	2

3. $(x-3)^2+(y+1)^2=49$



Center:	Radius:
(3, -1)	7

4. $(x+6)^2+(y+5)^2=16$



Center:	Radius:
(-6, -5)	4

Identify each part of the circle given its equation.

5. $(x-5)^2+(y-1)^2=64$ **Center:** (5, 1) **Radius:** 8	6. $(x+7)^2+(y-2)^2=324$ **Center:** (-7, 2) r = 18 **Diameter:** 36
7. $x^2+(y+10)^2=42.25$ **Center:** (0, -10) r = 6.5 **Diameter:** 13	8. $(x-6)^2+(y+11)^2=80$ **Center:** (6, -11) r = √80 **Radius:** 4√5
9. $(x+2)^2+(y+9)^2=196$ r = 14 **Center:** (-2, -9) C = 2π14 **Circumference:** 28π ; 88	10. $(x-3)^2+y^2=32$ r = 4√2 **Center:** (3, 0) C = 2π4√2 **Circumference:** 8π√2 ; 35.5
11. $(x-3)^2+(y+4)^2=81$ r = 9 **Center:** (3, -4) A = π9² **Area:** 81π ; 254.5	12. $(x+7)^2+(y+2)^2=108$ r = 6√3 **Center:** (-7, -2) A = π(6√3)² **Area:** 108π ; 339.3

GENERAL FORM EQUATION of a circle

The **general form** for the equation of a circle in the coordinate plane is given as:

$$Ax^2 + By^2 + Cx + Dy + E = 0$$

We can rewrite general form equations to standard form equations using a process called **completing the square.** Converting to standard form is useful when determining the center and radius of the circle.

Converting to STANDARD FORM

Use the steps below to convert the given equation to standard form. Then identify the radius and center of the circle.

Steps	Given: $x^2+y^2-2x+6y-6=0$
❶ Group the x-terms together and the y-terms together. Move the constant to the right side of the equation.	$x^2-2x+y^2+6y=6$
❷ Take half of the linear x-term and square it. Add this value to both sides of the equation.	$x^2-2x+1+y^2+6y=6+1$
❸ Take half of the linear y-term and square it. Add this value to both sides of the equation.	$x^2-2x+1+y^2+6y+9=7+9$
❹ You have created two perfect square trinomials. Rewrite them in factored form as squared binomials.	$(x-1)^2+(y+3)^2=16$
❺ Identify the center and radius of the circle.	center: (1, -3) radius: 4

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples

WRITING EQUATIONS *of Circles*

Recall: The standard form of the equation of a circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

where (h,k) is the center and r is the radius

Write each circle equation in standard form with the given information.

Set 1: Given the center and radius/diameter.

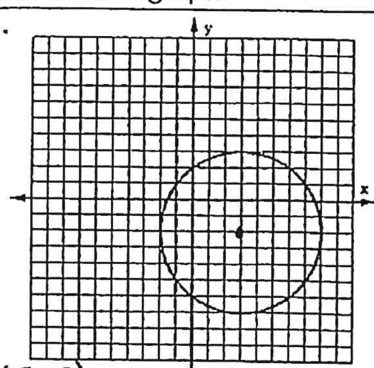
1. center: (5, 3), radius: 2 $(x-5)^2 + (y-3)^2 = 4$	**2.** center: (-1, 7), radius: 14 $(x+1)^2 + (y-7)^2 = 196$
3. center: (-2, -11), diameter: 18 $(r=9)$ $(x+2)^2 + (y+11)^2 = 81$	**4.** center: (0, 9), diameter: 32 $(r=16)$ $x^2 + (y-9)^2 = 256$
5. center: (-4, 0), radius: $\sqrt{47}$ $(x+4)^2 + y^2 = 48$	**6.** center: (15, -6), radius: $\sqrt{158}$ $(x-15)^2 + (y+6)^2 =$

Set 2: Given the center and circumference/area.

7. center: (-1, 4), circumference: 6π $(r=3)$ $(x+1)^2 + (y-4)^2 = 9$	**8.** center: (9, 12), circumference: 17π $(r=8.5)$ $(x-9)^2 + (y-12)^2 = 72.25$
9. center: (-8, -11), area: 16π $(r=4)$ $(x+8)^2 + (y+11)^2 = 16$	**10.** center: (2, -2), area: 361π $(r=19)$ $(x-2)^2 + (y+2)^2 = 361$

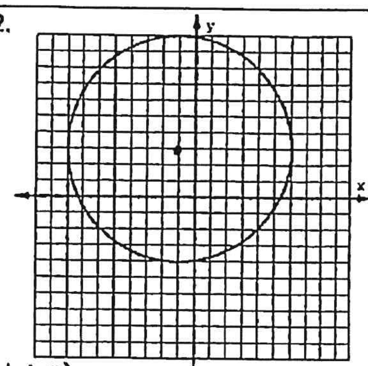
Set 3: Given a graph.

11.



$C(3,-2)$
$r = 5$
$(x-3)^2 + (y+2)^2 = 25$

12.



$C(-1,3)$
$r = 7$
$(x+1)^2 + (y-3)^2 = 49$

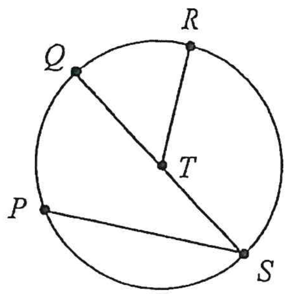
EXAMPLES	Write each equation in standard form. Then identify the indicated parts.
	13. $x^2 + y^2 + 14x - 4y + 17 = 0$; center and radius $x^2+14x+y^2-4y=-17$ $x^2+14x+\underline{49}+y^2-4y+\underline{4}=-17+\underline{49}+\underline{4}$ $(x+7)^2+(y-2)^2=36$ Center: (-7,2) ; radius: 6
	14. $x^2 + y^2 + 4x + 8y - 55 = 0$; center and radius $x^2+4x+y^2+8y=55$ $x^2+4x+\underline{4}+y^2+8y+\underline{16}=55+\underline{4}+\underline{16}$ $(x+2)^2+(y+4)^2=75$ Center: (-2,-4) ; radius: $5\sqrt{3}$
	15. $x^2 + y^2 - 12y - 85 = 0$; center and diameter $x^2+y^2-12y=85$ $x^2+y^2-12y+\underline{36}=85+\underline{36}$ $x^2+(y-6)^2=121$ center: (0,6) ; diameter: 22
	16. $x^2 + y^2 - 2y = 18x - 62$; center and diameter $x^2-18x+y^2-2y=-62$ $x^2-18x+\underline{81}+y^2-2y+\underline{1}=-62+\underline{81}+\underline{1}$ $(x-9)^2+(y-1)^2=20$ center: (9,1) ; diameter: $4\sqrt{5}$
	17. $x^2 + y^2 - 2x + 20 = 2x - 10y$; center and circumference $x^2+y^2-4x+10y+20=0$ $x^2-4x+\underline{4}+y^2+10y+\underline{25}=-20+\underline{4}+\underline{25}$ $(x-2)^2+(y+5)^2=9$ Center: (2,-5) ; circumfrence: 6π (18.8)
	18. $x^2 + 10x + 16y + 5 = 4x - y^2$; center and area $x^2+y^2+6x+16y+5=0$ $x^2+6x+\underline{9}+y^2+16y+\underline{64}=-5+\underline{9}+\underline{64}$ $(x+3)^2+(y+8)^2=68$ Center: (-3,-8) ; area: 68π (213.6)

Name: Review 1-11 NTI Day 9 **Geometry**

Date:__________ Per: ______ **Unit 10:** Circles

Quiz 10-1: Intro to Circles, Central Angles & Arcs, Arc Lengths

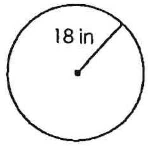
Use the circle below for questions 1 – 7.



1. Name a radius. 1. ________
2. Name a diameter. 2. ________
3. Name a chord that is not a diameter. 3. ________
4. Name a central angle. 4. ________
5. Name a minor arc. 5. ________
6. Name a major arc. 6. ________
7. Name a semicircle. 7. ________

For questions 8 and 9, find the area and circumference. Round to the nearest hundredth.

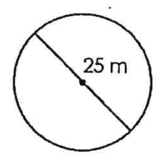
8.



A = ________

C = ________

9.



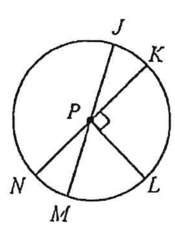
A = ________

C = ________

10. If the area of a circle is 201.06 cm², find its diameter and circumference. d = ________

C = ________

11. If $m\angle MPL = 63°$, find each measure.



a) $m\widehat{JK}$ = ______

b) $m\widehat{NJ}$ = ______

c) $m\widehat{JL}$ = ______

d) $m\widehat{KNM}$ = ______

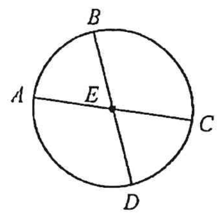
e) $m\widehat{MJL}$ = ______

f) $m\widehat{JLK}$ = ______

Review 12-16 NTI Day 10

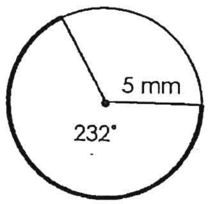
12. If $m\widehat{AD} = (19x - 49)^\circ$ and $m\widehat{CD} = (5x + 37)^\circ$, find $m\widehat{CD}$.

12. $m\widehat{CD}$ = ________

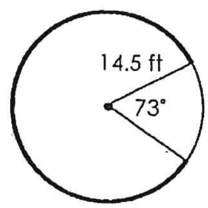


For questions 13 and 14, find the length of each bolded arc. Round to the nearest hundredth.

13.



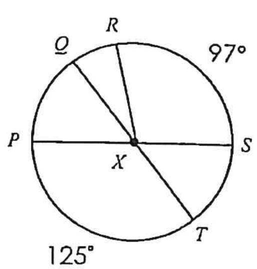
14.



13. ____________

14. ____________

15. Using the circle below, find each arc length. Round to the nearest hundredth.



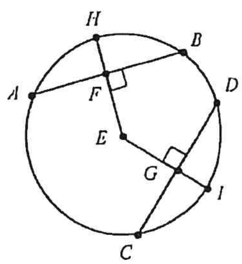
PS = 28 feet

a) $\widehat{ST}$ ____________ b) $\widehat{RPT}$ ____________

16. A merry-go-round has a radius of 18 feet. If a passenger gets on a horse located at the edge of the wheel and travels 38 feet, find the angle of rotation to the nearest degree.

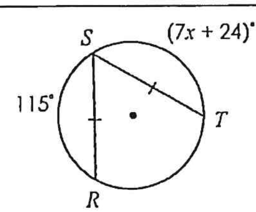
16. ____________

Name:	Date:
Topic:	Class:

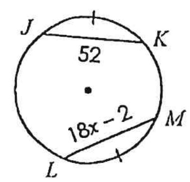
Main Ideas/Questions	Notes/Examples
Congruent **CHORDS & ARCS** 	• Two chords are congruent if and only if: a) ______________________________ ______________ ↔ ______________ b) ______________________________ ______________ ↔ ______________ • If a diameter or radius is ______________ to a chord, then it ________ the ________ and its ________. ________ → ________ and ________

Directions: Find the indicated value.

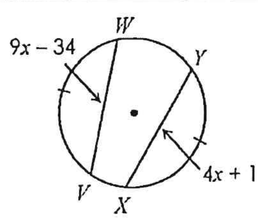
1. Find x.



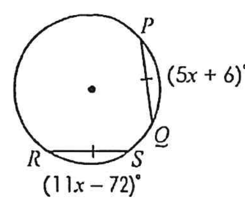
2. Find x.



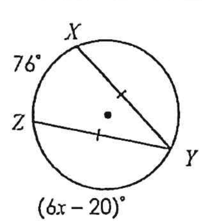
3. Find XY.



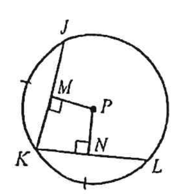
4. Find $m\widehat{RS}$.



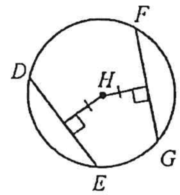
5. Find x.



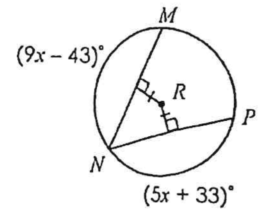
6. If $MP = 5x - 34$ and $PN = 2x - 4$, find MP.



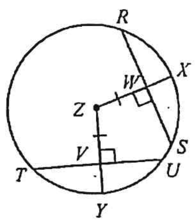
7. If $DE = 11x + 15$ and $FG = 32x - 27$, find DE.



8. Find $m\widehat{MP}$.



9. In circle Z, if $RS = 18$, and $m\widehat{TY} = 42°$, find each measure.



TV = ________

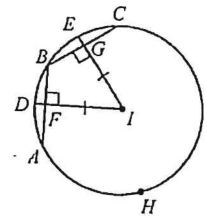
TU = ________

WS = ________

$m\widehat{YU}$ = ________

$m\widehat{RS}$ = ________

10. In circle I, if $BG = 17$, and $m\widehat{CHA} = 256°$, find each measure.



BC = ________

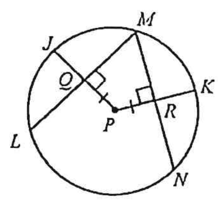
FB = ________

$m\widehat{AB}$ = ________

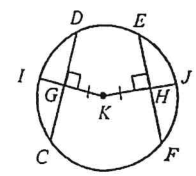
$m\widehat{BC}$ = ________

$m\widehat{EC}$ = ________

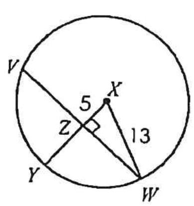
11. If $QM = 6x - 11$ and $MR = 2x + 9$, find MN.



12. If $m\widehat{CI} = (7x - 15)°$ and $m\widehat{EF} = (12x - 8)°$, find $m\widehat{CI}$.



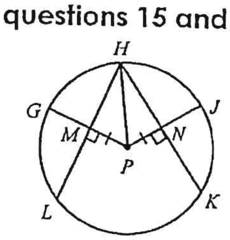
Use the circle below for questions 13 and 14.



13. Find VW.

14. Find $m\widehat{YW}$.

Use the circle below for questions 15 and 16.



$HK = 30$ and $PM = 8$

15. Find PH.

16. Find $m\widehat{GJ}$.

Name: NTIDay 11 1-5 only

Date: ________________ Per: ______

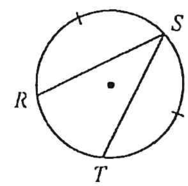
Unit 10: Circles

Homework 4: Congruent Chords & Arcs

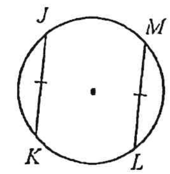
**** This is a 2-page document! ****

Directions: Find each value or measure.

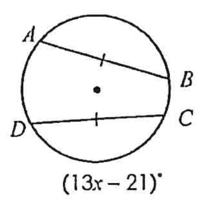
1. If $RS = 59$ and $ST = 10x - 31$, find x.



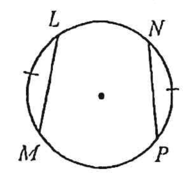
2. If $m\widehat{JK} = (7x - 39)^\circ$ and $m\widehat{ML} = 87^\circ$, find x.



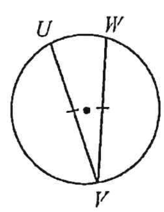
3. If $m\widehat{AD} = 85^\circ$ and $m\widehat{BC} = 31^\circ$, find the value of x.



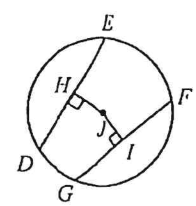
4. If $LM = 41 - 2x$ and $NP = 7x + 5$, find LM.



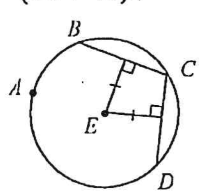
5. If $m\widehat{UV} = (8x - 17)^\circ$ and $m\widehat{WV} = (5x + 52)^\circ$, find $m\widehat{WV}$.



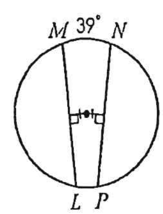
6. If $DE = GF$, $HJ = 3x + 20$ and $JI = 15x - 64$, find JI.



7. If $m\widehat{BC} = (9x - 53)^\circ$ and $m\widehat{CD} = (2x + 45)^\circ$, find $m\widehat{BAD}$.



8. If $m\widehat{LM} = (8x - 56)^\circ$ and $m\widehat{NP} = (5x + 22)^\circ$, find $m\widehat{LP}$.



Name: NTI Day 12 1-4

Date: ______________ Per: ______

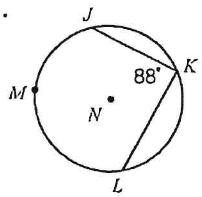
Unit 10: Circles

Homework 5: Inscribed Angles

** This is a 2-page document! **

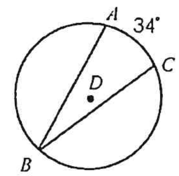
Directions: Find each angle or arc measure.

1.



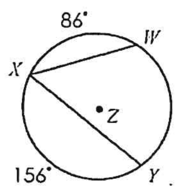
$m\widehat{JML}$ = ______

2.



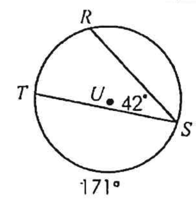
$m\angle ABC$ = ______

3.



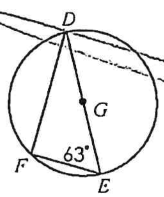
$m\angle WXY$ = ______

4.



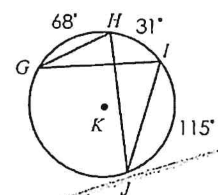
$m\widehat{RS}$ = ______

5.



$m\widehat{FE}$ = ______

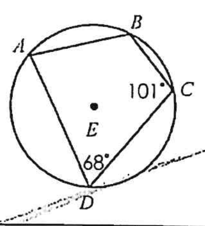
6.



$m\angle GHJ$ = ______

$m\angle GIJ$ = ______

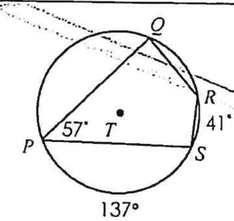
7.



$m\angle A$ = ______

$m\angle B$ = ______

8.



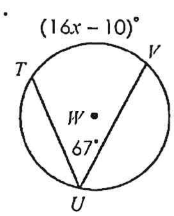
$m\angle Q$ = ______

$m\angle R$ = ______

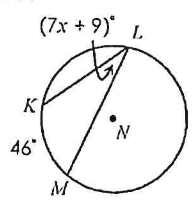
$m\angle S$ = ______

Directions: Find the value of x.

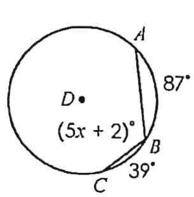
9.



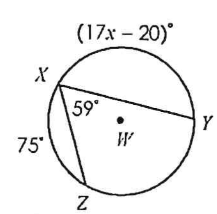
10.



11.



12.



NTI Day 13 9-12

Write each equation in standard form. Then identify each part.

13. $x^2 + y^2 + 8x - 6y - 25 = 0$		14. $x^2 + y^2 - 4x - 12y - 129 = 0$	
Standard Form:		Standard Form:	
Center:	Radius:	Center:	Radius:
15. $x^2 + y^2 + 14x + 10y + 73 = 0$		16. $x^2 + y^2 - 16x - 161 = 0$	
Standard Form:		Standard Form:	
Center:	Diameter:	Center:	Diameter:
17. $x^2 + y^2 = 24x + 4y - 85$		18. $x^2 + y^2 - 9x + 2y = x - 6y + 59$	
Standard Form:		Standard Form:	
Diameter:	Circumference:	Radius:	Area:

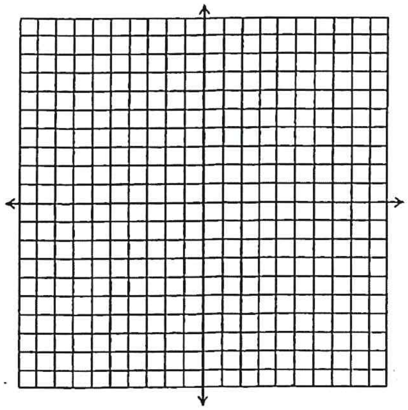
Name: NTI Day 14 1-4 **Unit 10:** Circles

Date: ______________________ Per: _______ **Homework 9:** Standard Form of a Circle

**** This is a 2-page document! ****

Directions: Graph each circle and identify its center and radius.

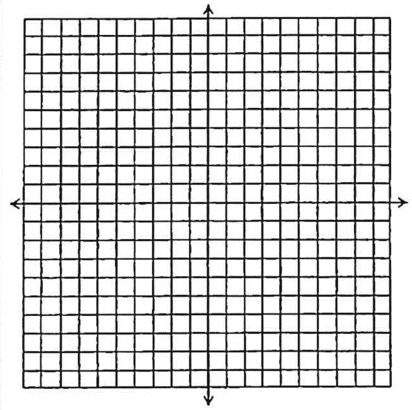
1. $(x-6)^2+(y-1)^2=9$



Center:

Radius:

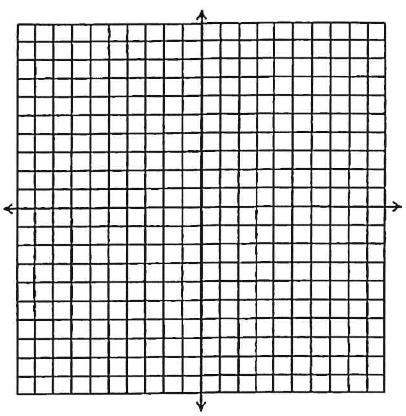
2. $(x-2)^2+(y+3)^2=36$



Center:

Radius:

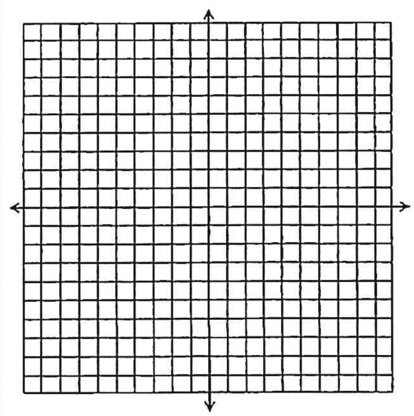
3. $x^2+(y+2)^2=64$



Center:

Radius:

4. $(x+4)^2+(y-5)^2=25$



Center:

Radius:

NTI Day 15 5-12 Gold

Directions: Identify each part of the circle given its equation.

5. $(x-9)^2+(y-4)^2=400$	**Center:**	**Radius:**
6. $(x+1)^2+(y-1)^2=256$	**Center:**	**Diameter:**
7. $(x+6)^2+y^2=90.25$	**Center:**	**Diameter:**
8. $(x-2)^2+(y+13)^2=150$	**Center:**	**Radius:**
9. $(x+7)^2+(y+4)^2=16$	**Center:**	**Circumference:**
10. $(x-10)^2+(y-5)^2=28$	**Radius:**	**Circumference:**
11. $x^2+(y+3)^2=81$	**Center:**	**Area:**
12. $(x+5)^2+(y-2)^2=135$	**Diameter:**	**Area:**

Name: NTI Day 15 1-6

Date: ______________ Per: ______

Unit 10: Circles

Homework 10: Equations of Circles

**** This is a 2-page document! ****

Directions: Use the information given to write the equation of the circle in standard form.

1. center: (2, 8), radius: 3	**2.** center: (5, -13), radius: 11
3. center (-9, -7), diameter: 34	**4.** center: (-8, 0), diameter: 5
5. center: (0, -16), radius: $\sqrt{15}$	**6.** center: (12, -1), radius: $\sqrt{209}$
7. center: (10, 7), circumference: 14π	**8.** center: (0, -1), circumference: 25π
9. center: (-4, -5), area: 400π	**10.** center: (-6, 2), area: π

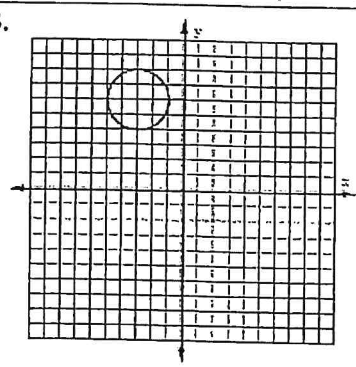
Use for questions 11-12: A pizzeria will only deliver to locations within a certain distance of the pizzeria. The delivery region has an area of 36π square miles.

11. Assuming the pizzeria is located at the origin, write an equation of the circle in which the pizzeria will deliver in standard form where x and y represent the distance from the pizzeria in miles.	**12.** Mikayla lives 4 miles north and 6 miles west of the pizzeria. Will the pizzeria deliver to her home?

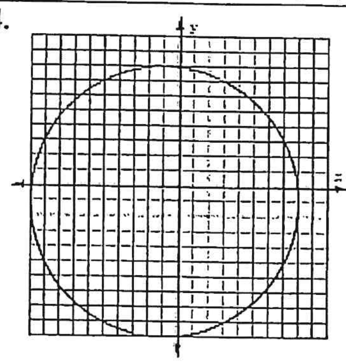
NTI Day 16 13-15

Directions: Write an equation for the circle shown on the graph in standard form.

13.



14.



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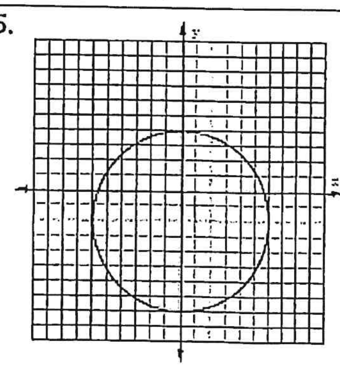


Directions: Use the information given to write the equation of the circle in standard form.

16. center: (-10, -4), point on circle: (4, -2)	**17.** center: (-9, -3), point on circle: (-11, 2)
18. center: (5, -7), point on circle: (-3, -1)	**19.** center: (8, 2), point on circle: (14, -1)
20. endpoints of diameter: (13, -1) and (-15, 9)	**21.** endpoints of diameter: (6, -2) and (10, -10)
22. endpoints of diameter: (-8, 12) and (0, 2)	**23.** endpoints of diameter: (8, -1) and (-2, 3)

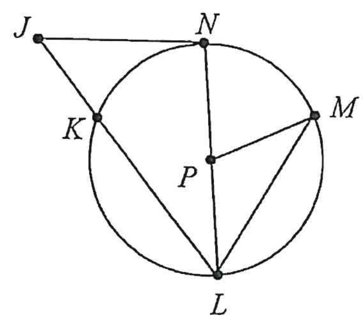
Unit 10 Test Study Guide
(Circles)

Name: NTI Days 17-20

Date: ____________________ Per: ________

Topic 1: Parts of Circles

1. Using the diagram below, give an example of each circle part.



a. Center: ________

b. Radius: ________

c. Diameter: ________

d. Chord: ________

e. Secant: ________

f. Tangent: ________

g. Central Angle: ________

h. Inscribed Angle: ________

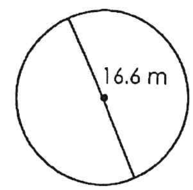
i. Minor Arc: ________

j. Major Arc: ________

k. Semicircle: ________

Topic 2: Area & Circumference

2. Find the area and circumference of the circle to the right.



3. Find the radius of a circle with a circumference of 106.81 centimeters.

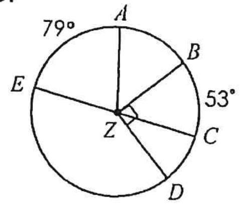
4. Find the diameter of a circle with an area of 95.03 square feet.

5. Find the circumference of a circle with an area of 254.47 square inches.

6. Find the area of a circle with a circumference of 30π meters.

Topic 3: Central Angles

7. Find each arc measure.



a) $m\widehat{CD}$ = ________

b) $m\widehat{AB}$ = ________

c) $m\widehat{ED}$ = ________

d) $m\widehat{EB}$ = ________

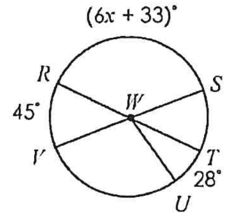
e) $m\widehat{BDE}$ = ________

f) $m\widehat{DEC}$ = ________

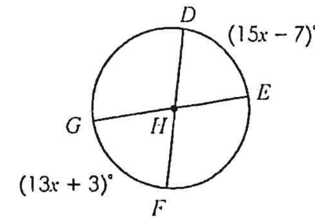
NTI Day 17 1-6

NTI Day 18 7-13

8. Solve for x.

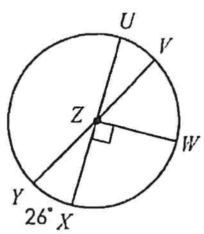


9. Find $m\widehat{EF}$.



Topic 4: Arc Lengths

If the circle below has a radius of 15 cm, find each arc length.

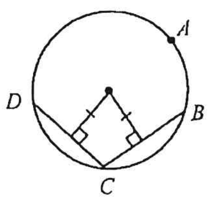


10. $\widehat{VW}$

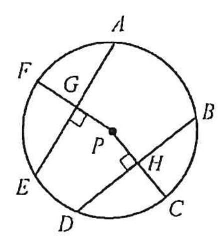
11. $\widehat{UXV}$

Topic 5: Chords & Arcs

12. If $m\widehat{DC} = (12x + 7)°$ and $m\widehat{CB} = (18x - 23)°$, find $m\widehat{DAB}$.



13. If $GP = PH$, $GA = 17$, $m\widehat{ED} = 37°$, and $m\widehat{AB} = 87°$, find each measure.



DB = ________

EG = ________

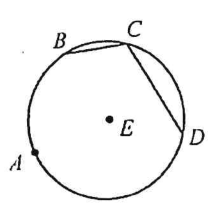
$m\widehat{DB}$ = ________

$m\widehat{FA}$ = ________

$m\widehat{DC}$ = ________

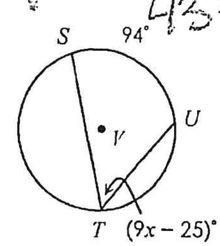
Topic 6: Inscribed Angles

16. If $m\angle BCD = (7x + 10)°$ and $m\widehat{BAD} = (19x - 50)°$, find $m\widehat{BAD}$.

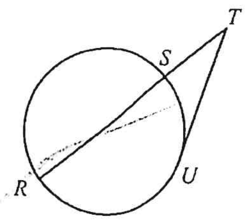


NTI Day 19 15-16 43-44

15. Solve for x.



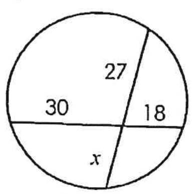
36. If $m\widehat{RU} = (16x - 13)^\circ$, $m\widehat{SU} = (11x - 24)^\circ$, and $m\angle STU = (3x + 1)^\circ$, find $m\widehat{SU}$.



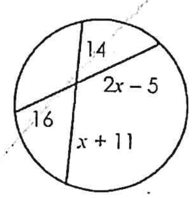
Topic 9: Segment Lengths formed by Intersecting Chords, Secants, & Tangents

For questions 37-40, solve for x. Assume segments that appear to be tangent are tangent.

37.



38.



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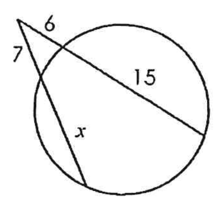
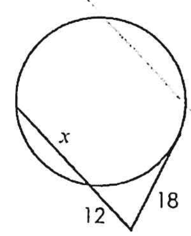
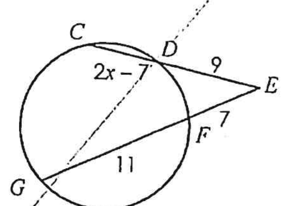


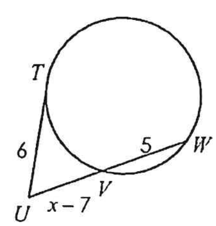
40.



41. Find CD.



42. Find UW.



Topic 10: Equations of Circles

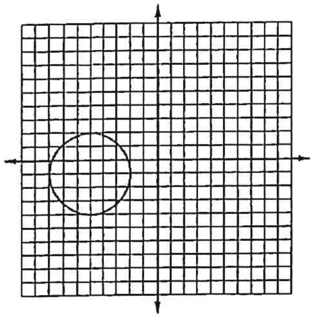
Identify the center and radius/diameter for the following circles.

43. $(x+2)^2 + (y-7)^2 = 16$	**Center:**	**Radius:**
44. $x^2 + (y+6)^2 = 121$	**Center:**	**Diameter:**

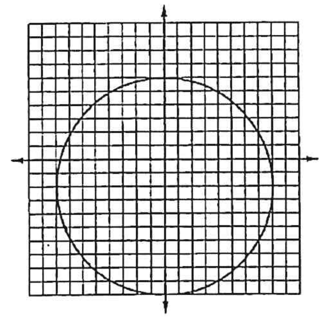
NTI Day 20 45-50

Write the equation of the circle in standard form shown on the graph.

45.



46.



Write an equation of the circle in standard form with the given characteristics.

47. center: (-3, 4), radius: 7

48. center: (-9, 0), diameter: 20

49. center: (-7, -1), diameter: 9

50. center: (12, 5), radius: $\sqrt{89}$

51. center: (2, -2), circumference: 12π

52. center: (0, 10), area: 225π

53. center: (-4, 7), point on circle: (-1, 9)

54. endpoints of diameter: (-18, -3) and (8, 11)

Write each equation in standard form. Then identify the center and radius.

55. $x^2 + y^2 - 8x + 12y + 34 = 0$

56. $x^2 + y^2 + x = 279 + 3x - 6y$

Alg 2 Travis

Arithmetic Sequences **Notes for March 16-18th**

Sequence – an ordered set of numbers. The dots (ellipsis) mean there are more numbers. A sequence that does not end is called **infinite.**

EX. 3, 5, 7, 9, . . .

Some sequences have a rule to describe the n th term. The rule for the sequence above is $a_n = 2n + 1$.

Nth term – the general term. The letter k is sometimes used instead of n.

Finite Sequence has a certain number of terms.

Term of a Sequence – each number in a sequence

Explicit Rule or Formula – makes reference to a first term. It describes the nth term of a sequence using the number n.

EX. 2, 4, 6, 8, 10, . . . , $a_n = 2n$

Recursive Rule or Formula – makes reference to a previous term.

EX. $a_1 = 6$

$a_n = a_{n-1} - 4$, for $n > 1$

Generating a Sequence Using an Explicit Formula

EX #1: A sequence has an explicit formula $a_n = 12n + 3$. Find term a_{12}.

EX #2: A sequence is given by $a_n = 4 + (n-1)5$

A. Find the first three terms.

B. Find the 11th term.

Finding General Terms

EX #3: The numbers in the sequences below follow a general pattern. Can you write the general rule?		
A. 6, 16, 26, 36, . . .	**B.** 1, 1.25, 1.5, 1.75, . . .	**C.** $-1, -4, -7, -10, -13, \ . \ . \ .$

Recursive Definitions for a Sequence

RECURSIVE FORMULA DEFINITION

Recursive sequences contain two parts.

1.) An initial condition (The value of the first term.) $a_1 = k$

2.) The recursive formula: $a_n = a_{n-1} + d$ for all $n > 1$

EX #4: Look for simple addition or subtraction patterns to relate consecutive terms. Write a recursive rule for the sequence.

A. $1, 4, 7, 10, 13, \cdots$	**B.** $4, -1, -6, -11, -16, \cdots$

Arithmetic Sequences

Arithmetic sequences have the same differences between terms (add):
$d = a_2 - a_1,\ a_3 - a_2,\ \ldots$

d = common difference between consecutive terms

Each term in an arithmetic sequence can be obtained recursively from its previous term by adding d:

EXPLICIT or GENERAL FORM:	$a_n = a_1 + (n-1)d$ For $n \geq 1$
	$a_n = dn + c$ $c = a_1 - d$
RECURSION FORMULAS:	$a_1 = k$ $a_{n+1} = a_n + d$ (for all $n \geq 1$)
	$a_1 = k$ $a_n = a_{n-1} + d$ (for all $n > 1$)

Analyzing Arithmetic Sequences

EX#5: Is the sequence an arithmetic sequence? 1,5,9,13,17,...

EX #6: Find the 46th term of the sequence 2,5,8,11,14,...

EX #7: Find the missing numbers in the sequence 80, ☐ , ☐ ,125,...

Using Explicit Formulas

EX #8: The number of seats in the first 16 rows of an arena form an arithmetic sequence. If there are 20 seats in Row 1, 23 seats in Row 2, how many seats are in Row 16?

EX #9: The Arithmetic Mean

Find the missing term of the arithmetic sequence: ...132 , ☐ ,98,...

Notes Key for March 16-18

Arithmetic Sequences

Sequence – an ordered set of numbers. The dots (ellipsis) mean there are more numbers. A sequence that does not end is called **infinite.**

EX. 3, 5, 7, 9, . . .

Some sequences have a rule to describe the n^{th} term. The rule for the sequence above is $a_n = 2n + 1$.

Nth term – the general term. The letter k is sometimes used instead of n.

Finite Sequence has a certain number of terms.

Term of a Sequence – each number in a sequence

Explicit Rule or Formula – makes reference to a first term. It describes the nth term of a sequence using the number n.

EX. 2, 4, 6, 8, 10, . . . , $a_n = 2n$

Recursive Rule or Formula – makes reference to a previous term.

EX. $a_1 = 6$

$a_n = a_{n-1} - 4$, for $n > 1$

Generating a Sequence Using an Explicit Formula

EX #1: A sequence has an explicit formula $a_n = 12n + 3$. Find term a_{12}.

$a_{12} = 12(12) + 3$

$a_{12} = 144 + 3$

$a_{12} = 147$

EX #2: A sequence is given by $a_n = 4 + (n-1)5$

A. Find the first three terms.

$a_1 = 4$

$a_2 = 4 + (2-1)(5) = 9$

$a_3 = 4 + (3-1)(5) = 14$

4, 9, 14

B. Find the 11th term.

$a_{11} = 4 + (11-1)(5)$

$a_{11} = 4 + (10)(5)$

$a_{11} = 4 + 50$

$a_{11} = 54$

Finding General Terms

EX #3: The numbers in the sequences below follow a general pattern. Can you write the general rule?

A. 6, 16, 26, 36, . . .	B. 1, 1.25, 1.5, 1.75, . . .	C. −1, −4, −7, −10, −13, . . .
difference $d=10$	difference $d=\frac{1}{4}$	difference $d=-3$
Rule:	Rule:	Rule:
$a_n = 6+(n-1)(10)$	$a_n = 1+(n-1)(0.25)$	$a_n = -1+(n-1)(-3)$
$a_n = 10n-4$	$a_n = .25n+.75$	$a_n = -3n+2$

Recursive Definitions for a Sequence

RECURSIVE FORMULA DEFINITION

Recursive sequences contain two parts.

*1.) An initial condition (The value of the first term.) $a_1 = k$

* 2.) The recursive formula: $a_n = a_{n-1} + d$ for all $n > 1$

EX #4: Look for simple addition or subtraction patterns to relate consecutive terms. Write a recursive rule for the sequence.

A. $1, 4, 7, 10, 13, \cdots$

$d = 3$ difference

$$\begin{cases} a_1 = 1 \\ a_n = a_{n-1} + 3 \end{cases}$$

B. $4, -1, -6, -11, -16, \cdots$

$d = -5$ difference

$$\begin{cases} a_1 = 4 \\ a_n = a_{n-1} - 5 \end{cases}$$

Arithmetic Sequences

Arithmetic sequences have the same differences between terms (add):

$d = a_2 - a_1,\ a_3 - a_2,\ \ldots$

d = common difference between consecutive terms

Each term in an arithmetic sequence can be obtained recursively from its previous term by adding d:

EXPLICIT or GENERAL FORM:	$a_n = a_1 + (n-1)d$ For $n \geq 1$
	$a_n = dn + c$ $c = a_1 - d$
RECURSION FORMULAS:	$a_1 = k$ $a_{n+1} = a_n + d$ (for all $n \geq 1$)
	$a_1 = k$ $a_n = a_{n-1} + d$ (for all $n > 1$)

Analyzing Arithmetic Sequences

EX#5: Is the sequence an arithmetic sequence? 1,5,9,13,17,...

$d = 4$

yes, common difference is 4

rule:

$a_n = 4n - 3$

$c = a_1 - d$

$c = 1 - 4 \Rightarrow -3$

EX #6: Find the 46th term of the sequence 2,5,8,11,14,...

model:

$a_n = a_1 + (n-1)d$

$d = 3$

$a_1 = 2$

$a_{46} = 2 + (46-1)(3)$

$a_{46} = 2 + (45)(3)$

$a_{46} = 2 + 135$

$a_{46} = 137$

EX #7: Find the missing numbers in the sequence 80, ☐, ☐, 125,...

3 spaces between 80 and 125

$d = 125 - 80$

$d = 45$

$3\overline{)45}$ = 15

80, <u>95</u>, <u>110</u>, 125

Using Explicit Formulas

EX #8: The number of seats in the first 16 rows of an arena form an arithmetic sequence. If there are 20 seats in Row 1, 23 seats in Row 2, how many seats are in Row 16?

$R_1 = 20$

$R_2 = 23$

$R_{16} = ?$

common difference 3

Rule: $a_n = a_1 + (n-1)d$

$a_n = 20 + (n-1)(3)$

$a_n = 20 + 3n - 3$

$a_n = 3n + 17$

$a_{16} = 3(16) + 17$

$a_{16} = 48 + 17$

$a_{16} = 65$ seats in Row 16

EX #9: The Arithmetic Mean

Find the missing term of the arithmetic sequence: ...132, ☐, 98,...

average = mean

$$\frac{132 + 98}{2} = 115$$

Geometric Sequences Notes for March 19-23rd

A sequence with a constant ratio between consecutive terms is known as a ______________________ sequence.

A **geometric sequence** with a starting value **a** and a common ratio ***r*** is a sequence of the form $a, ar, ar^2, ar^3, \ldots$

EX. 4, 8,16, 32, 64, . . .

The ratio of consecutive terms of a geometric sequence is called a ______________________ ratio.

EX. 4, 8,16, 32, 64, . . . has a common ratio of __________.

Given two positive numbers, the positive square root of the product of the two numbers is the ______________________

EX. The geometric mean of 3 and 48 is ________________.

Geometric Sequence Formulas

EXPLICIT DEFINITION:	$a_n = a_1 \cdot r^{n-1}$, for $n \geq 1$
RECURSIVE DEFINITION:	$a_1 = a$ $a_n = a_{n-1} \cdot r$ for $n > 1$

How to find the ratios between consecutive terms?

EX. 2,4,8,16,...

Identifying Geometric Sequences

EX #1: Determine whether each sequence is geometric. If so, find the common ratio, r.

A. 3, 6, 12, 24, . . .	**B.** $3, -9, 27, -81, \ldots$
C. 1, 5, 9, 13, 17, . . .	**D.** $48, -12, 3, \ldots$

Analyzing Geometric Sequences

EX #2: Find the indicated terms of the geometric sequences.

A. Find the 6th term of the sequence $3, -15, 75, \ldots$	**B.** Find the 11th term of the sequence $64, -32, 16, \ldots$

Using a Geometric Sequence

EX #3: When radioactive substances decay, the amount remaining will form a geometric sequence when measured over constant intervals of time. The table shows the amount of *Adamsium* (Ad) is a highly radioactive element, initially, and after 2 hours. Find the amounts left after 1 hour, 3 hours, and 4 hours.

Hours Lapsed	0	1	2	3	4
Grams of *Adamsium*	1320		384		

Using the Geometric Mean

The geometric mean of two positive numbers x and *y* is ___________ .

In a sequence the formula is:

In a geometric sequence, the square of the middle term of any three consecutive terms is equal to the product of the other two terms. For example, $4, -8, 16, -32, \ldots$

EX #4: Insert the **geometric mean** between the following.	
A. 45 ,□ ,80,...	**B.** 972,□ ,□ ,□ ,12...

Writing Formulas for Sequences

EX #5: Write an explicit formula for the sequence and generate the first five terms.	
A. $a_1 = 12,\ r = -\frac{1}{3}$	**B.** $a_1 = 5$,$r = 0.2$

Notes Key for March 19 - 23rd

Geometric Sequences

A sequence with a constant ratio between consecutive terms is known as a geometric sequence.

A **geometric sequence** with a starting value **a** and a common ratio **r** is a sequence of the form $a, ar, ar^2, ar^3, \ldots$

EX. 4, 8, 16, 32, 64, . . .

The ratio of consecutive terms of a geometric sequence is called a common ratio.

EX. 4, 8, 16, 32, 64, . . . has a common ratio of 2.

Given two positive numbers, the positive square root of the product of the two numbers is the geometric mean

EX. The geometric mean of 3 and 48 is 12. $\sqrt{3 \cdot 48}$ $\sqrt{144}$

Geometric Sequence Formulas

EXPLICIT DEFINITION:	$a_n = a_1 \circ r^{n-1}$, for $n \geq 1$
RECURSIVE DEFINITION:	$a_1 = a$ $a_n = a_{n-1} \circ r$ for $n > 1$

How to find the ratios between consecutive terms?

EX. 2, 4, 8, 16, ...

1) Check $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = ?$

$r = \frac{4}{2} = \frac{8}{4} = \frac{16}{8}$

$r = 2$

Identifying Geometric Sequences

EX #1: Determine whether each sequence is geometric. If so, find the common ratio, r.

A. 3, 6, 12, 24, . . .

Rule: $a_n = a_1 \cdot r^{n-1}$

$r = 2$

$a_n = 3(2)^{n-1}$

B. 3, –9, 27, –81,...

$r = -3$

$a_n = 3(-3)^{n-1}$

Danger: -3^{n-1} and $(-3)^{n-1}$ are not the same!

C. 1, 5, 9, 13, 17, . . .

$r = \frac{5}{1} \neq \frac{9}{5} \neq \frac{13}{9}$

This sequence is not geometric

D. 48, –12, 3,...

$r = -\frac{1}{4}$

$a_n = 48\left(-\frac{1}{4}\right)^{n-1}$

Analyzing Geometric Sequences

EX #2: Find the indicated terms of the geometric sequences.

A. Find the 6th term of the sequence

3, –15, 75,...

1) $r = -5$

2) $a_n = 3(-5)^{n-1}$

3) $a_6 = 3(-5)^{6-1}$

$a_6 = 3(-5)^5$

$a_6 = 3(-3125)$

$a_6 = -9375$

B. Find the 11th term of the sequence

64, –32, 16,...

1) $r = -\frac{1}{2}$

2) $a_n = 64\left(-\frac{1}{2}\right)^{n-1}$

3) $a_{11} = 64\left(-\frac{1}{2}\right)^{11-1}$

$a_{11} = 64\left(-\frac{1}{2}\right)^{10}$

$a_{11} = 64\left[\frac{1}{1024}\right]$

$a_{11} = \frac{1}{16}$

Using a Geometric Sequence

EX #3: When radioactive substances decay, the amount remaining will form a geometric sequence when measured over constant intervals of time. The table shows the amount of *Adamsium* (Ad) is a highly radioactive element, initially, and after 2 hours. Find the amounts left after 1 hour, 3 hours, and 4 hours.

Hours Lapsed	0	1	2	3	4
Grams of *Adamsium*	1320	712	384	207	112

1) Use mean $\sqrt{a_0 \cdot a_2}$

$a_1 = \sqrt{1320 \cdot 384}$

$a_1 \approx 711.95$

2) Find ratio

$r = \frac{712}{1320}$

$r \approx 0.5\overline{39}$

3) Use "r" as a "multiplier."

$a_3 = 384(0.53939)$

$a_3 \approx 207.13$

$a_4 = 207(0.53939)$

$a_4 \approx 111.65$

Using the Geometric Mean

The geometric mean of two positive numbers x and y is $\sqrt{xy}$.

In a sequence the formula is:

$$\text{Mean} = a_n = \sqrt{a_{n-1} \cdot a_{n+1}}$$

In a geometric sequence, the square of the middle term of any three consecutive terms is equal to the product of the other two terms. For example, 4, –8, 16, –32, …

4   –8   16   –32

$\sqrt{4(16)} = \pm 8$ $\sqrt{(-8)(-32)} = \pm 16$

EX #4: Insert the **geometric mean** between the following.	
A. 45, ☐, 80,... $\sqrt{45\cdot 80}$ $\sqrt{3600}$ 60	B. 972, ☐, ☐, ☐, 12... 1) Find middle term $\sqrt{972\cdot 12} = 108$ 2) Find second term $\sqrt{972\cdot 108} = 324$ 3) Find fourth term $\sqrt{108\cdot 12} = 36$ 972, 324, 108, 36, 12, ...

Writing Formulas for Sequences

EX #5: Write an explicit formula for the sequence and generate the first five terms.	
A. $a_1 = 12,\ r = -\frac{1}{3}$ $a_n = a_1 \cdot r^{n-1}$ $a_n = 12\left(-\frac{1}{3}\right)^{n-1}$ $a_1 = 12$ $a_2 = -4$ $a_3 = 4/3$ $a_4 = -4/9$ $a_5 = 4/27$	B. $a_1 = 5,\ r = 0.2$ $a_n = a_1 \cdot r^{n-1}$ $a_n = 5(0.2)^{n-1}$ $a_1 = 5$ $a_2 = 1.0$ $a_3 = 0.2$ $a_4 = 0.04$ $a_5 = 0.008$

Arithmetic Series Notes for March 26-30th

Vocabulary	Example:
The sum of the terms of a sequence is called a ____________.	The series 3 + 6 + 9 + 12 + 15 corresponds to the sequence 3, 6, 9, 12, 15. The sum of the series is _______.
An __________________ series has infinitely many terms.	
Any series whose terms form an arithmetic sequence is known as an ____________________________.	1 + 5 + 9 + 13 + 17 + 21 is an arithmetic series with ________ terms.
______________________________ is the sum of the terms of the sequence denoted as $\sum_{k=1}^{n} a_k$, where ______ is the summation of notation.	$\sum_{n=1}^{5}(3n-2)$
In summation notation, the least and greatest integer values of the index n are called _______________.	$\sum_{n=1}^{5}(3n-2)$ In this example, the limits are _____ and _____ .

Sum of an Arithmetic Sequence

When a_1 is the first term, a_n is the n th term, and n is the number of terms.	
Formula:	
When the first and last terms are known:	$S_n = \frac{n}{2}(a_1 + a_n)$
When a_n is not known:	$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$

Summation Notation

The Greek letter, sigma, $\sum$ is used to indicate a sum. Use **limits** (upper bound and lower bound) to indicate how many terms you are adding. Write the **limits** below and above the $\sum$ to show the first and last terms of the series.

$$\sum_{n=2}^{6} n^2$$

Finding the Sum of a Finite Sequence

EX #1: Find the sum of the first 100 natural numbers.

EX #2: Find the sum of the first 15 terms of the arithmetic series $2+5+8+11+14+17+\ldots$

Finding the Sum of a Series

EX #3: Find the sum of the finite arithmetic series $4+9+14+19+24+\ldots+99$

EX #4: Find the sum of the series $\sum_{k=1}^{13}(4k+5)$

Writing Summation Notation

EX #5: Write Sigma Notation for the series:

A. $3+6+9+12+15$

B. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$

Series and Summation

EX#6: If an arithmetic series has $a_1 = 2, d = 5,$ and $n = 20$.

A. Write the summation notation for the series.

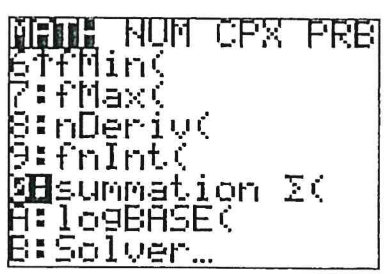
B. Find S_n of the series.

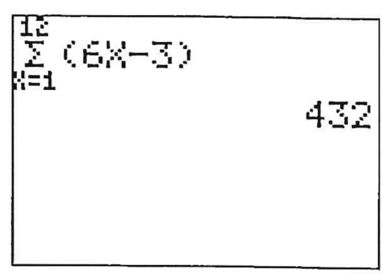
Using the Sum of a Finite Series

EX #7: How many logs will be in a pile of timbered trees if there are 30 logs in the bottom layer, 29 in the second, and so on until there is one in the top layer?

EX #8: Use a Calculator to find the sum of the series

$$\sum_{k=1}^{12}(6k-3)$$





Notes Key for March 26-30th

Arithmetic Series

Vocabulary	Example:
The sum of the terms of a sequence is called a series.	The series 3 + 6 + 9 + 12 + 15 corresponds to the sequence 3, 6, 9, 12, 15. The sum of the series is ______.
An infinite series has infinitely many terms.	$2+5+8+11+14+\ldots$
Any series whose terms form an arithmetic sequence is known as an arithmetic series.	1 + 5 + 9 + 13 + 17 + 21 is an arithmetic series with 6 terms.
Summation notation is the sum of the terms of the sequence denoted as $\sum_{k=1}^{n} a_k$, where K is the summation of notation.	$\sum_{n=1}^{5}(3n-2)$ $1+4+7+10+13$ 35
In summation notation, the least and greatest integer values of the index n are called limits.	$\sum_{n=1}^{5}(3n-2)$ (5 ← upper limit; $n=1$ ← lower limit) In this example, the limits are 1 and 5.

Sum of an Arithmetic Sequence

When a_1 is the first term, a_n is the n th term, and n is the number of terms.	
Formula:	
When the first and last terms are known:	$S_n = \frac{n}{2}(a_1 + a_n)$
When a_n is not known:	$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$

Summation Notation

The Greek letter, sigma, $\sum$ is used to indicate a sum. Use **limits** (upper bound and lower bound) to indicate how many terms you are adding. Write the **limits** below and above the $\sum$ to show the first and last terms of the series.

$\sum_{n=2}^{6} n^2 = 2^2 + 3^2 + 4^2 + 5^2 + 6^2$

$= 4 + 9 + 16 + 25 + 36$

$= \underline{90}$

add $\rightarrow \sum_{n=2}^{6} n^2$ (6 ← stop; $n=2$ ↑ start; n^2 ← explicit rule)

Finding the Sum of a Finite Sequence

EX #1: Find the sum of the first 100 natural numbers.

$a_1 = 1$

$a_{100} = 100$

$S_{100} = \frac{100}{2}[1 + 100]$

$= 50[101]$

$= \underline{5{,}050}$

EX #2: Find the sum of the first 15 terms of the arithmetic series $2 + 5 + 8 + 11 + 14 + 17 + \ldots$

When we don't know a_n rule use:

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$a_1 = 2$
$d = 3$
$n = 15$

$S_{15} = \frac{15}{2}[2(2) + (15-1)(3)]$

$= \frac{15}{2}[4 + 42]$

$= \frac{15}{2}[46]$ (46 cancelled to 23)

$S_{15} = \underline{345}$

Finding the Sum of a Series

EX #3: Find the sum of the finite arithmetic series 4+9+14+19+24+ . . .+99

Sequence 4, 9, 14, 19, ..., 99

rule: $a_n = 5n - 1$

$99 = 5n - 1$

$100 = 5n$

$n = 20$ ← terms

$S_{20} = \frac{20}{2}[4 + 99]$

$S_{20} = 10[103]$

$S_{20} = 1030$

How many terms in series?

EX #4: Find the sum of the series $\sum_{k=1}^{13}(4k+5)$

1) $n = 13$

2) $a_1 = 4(1)+5$
$a_1 = 9$

3) $a_{13} = 4(13)+5$
$a_{13} = 57$

4) $S_{13} = \frac{13}{2}[9+57]$
$S_{13} = \frac{13}{2}[66]$ (66 crossed out, 33; 2 crossed out)
$S_{13} = 429$

Writing Summation Notation

EX #5: Write Sigma Notation for the series:

A. $3+6+9+12+15$

Sequence: 3, 6, 9, 12, 15

rule: $3n$ explicit rule

$n=1$ lower

5 terms upper

$$\sum_{n=1}^{5}[3n]$$

B. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$

$\frac{1}{1+1}, \frac{2}{2+1}, \frac{3}{3+1}, \ldots, \frac{n}{n+1}$

$n=1$; $a_1 = \frac{1}{2}$; 6 terms

$$\sum_{n=1}^{6}\left[\frac{n}{n+1}\right]$$

Series and Summation

EX#6: If an arithmetic series has $a_1 = 2, d = 5$, and $n = 20$.

A. Write the summation notation for the series.

lower limit = 1
upper limit = 20
explicit rule $5n - 3$

$$\sum_{n=1}^{20} (5n-3)$$

B. Find S_n of the series.

$$S_{20} = \frac{20}{2}[2+97]$$
$$= 10(99)$$
$$S_{20} = 990$$

$a_1 = 2$
$a_{20} = 5(20) - 3$
$a_{20} = 97$

Using the Sum of a Finite Series

EX #7: How many logs will be in a pile of timbered trees if there are 30 logs in the bottom layer, 29 in the second, and so on until there is one in the top layer?

How many layers? $30 = n$
Sequence $30, 29, 28, \ldots, 1$

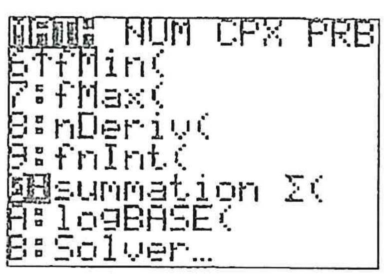
$$S_{30} = \frac{30}{2}[1+30]$$
$$= 15(31)$$
$$= 465 \text{ logs}$$

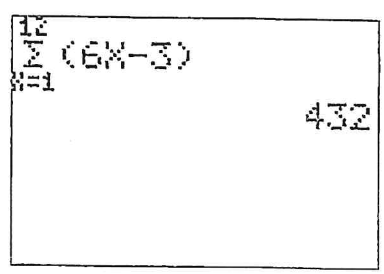
EX #8: Use a Calculator to find the sum of the series

$$\sum_{k=1}^{12} (6k-3)$$

MATH > 0

$$\sum_{\square = \square}^{\square} (\square)$$





Geometric Series Notes for March 30 - April 2nd

Vocabulary	Example:
The sum of the terms in a geometric sequence is known as a ______________________.	The series $2.5+5+10+20+40$ corresponds to the sequence $2.5, 5, 10, 20, 40$. The sum of the series is 77.5.
An infinite series $a_1 + a_2 + \ldots + a_n + \ldots$ is said to ______________ if the sum $a_1 + a_2 + \ldots + a_n$ gets closer and closer to a real number as n increases.	
An infinite series will ______________ if it does not converge.	

Sums of Geometric Series

When a_1 is the first term, r is the common ratio, $r \neq 1$, and n is the number of terms.	
Formula:	
Sum of a Finite Geometric Sequence with ratio $r \neq 1$	$S_n = a_1\left(\dfrac{1-r^n}{1-r}\right)$
Alternate Form of the Finite Formula	$S_n = \dfrac{a_1 - a_1 \cdot r^n}{1-r}$
Sum of an Infinite Geometric Series if $\lvert r \rvert < 1$	$S = \dfrac{a_1}{1-r}$
NOTE: An infinite geometric series with $\lvert r \rvert \geq 1$ does not have a finite sum. The series is said to **diverge**.	

EX #1: What do you need to find the sum of the first 8 terms in the series $3+6+12+24+\ldots$?

Finding the Sum of a Finite Series

EX #2: Find the sum of the finite geometric series

A. $3 + 6 + 12 + 24 + \ldots + 1536$

B. $-15 + 30 - 60 + 120 - 240 + 480$

EX #3: Find the sum of the finite geometric series.

A. $\sum_{n=1}^{10} 5(-2)^{n-1}$

B. $\sum_{k=0}^{24} 3\left(\frac{1}{2}\right)^{n}$

Using the Geometric Series Formula

EX #5: You make a contract with your parents to work eight hours every day during the month of July. The first day you will be paid one penny, the second day two cents, on the third day you get 4 cents, the fourth day 8 cents, and so on. Is this a good plan for you or do your parents get the most out of the contract?

Series and Summation

EX#6: Write the summation notation and find the sum of the first 6 terms of the geometric series $3+15+75+375+\ldots$

EX #7: Write the summation notation and find the sum of the first 10 terms of the geometric series $2-1+\frac{1}{2}-\frac{1}{4}+\ldots$

Analyzing Infinite Geometric Series

EX #8: Determine whether the series *converges* or *diverges*. If it converges, find the sum.

A. $\frac{1}{2}+\frac{3}{4}+\frac{9}{8}+\ldots$

B. $\frac{1}{3}-\frac{1}{9}+\frac{1}{27}-\frac{1}{81}+\ldots$

C. If the series converges can you use a calculator to find the sum?

$$\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^{n}$$

Notes Key for March 30–April 2nd

Geometric Series

Vocabulary	Example:
The sum of the terms in a geometric sequence is known as a geometric series.	The series $2.5+5+10+20+40$ corresponds to the sequence $2.5, 5, 10, 20, 40$. The sum of the series is 77.5.
An infinite series $a_1+a_2+\ldots+a_n+\ldots$ is said to converge if the sum $a_1+a_2+\ldots+a_n$ gets closer and closer to a real number as n increases.	$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$ $r=\frac{1}{2}$, series converges to 2
An infinite series will diverge if it does not converge.	$1+2+4+8+\ldots$ diverges since $r=2$ the sum increases

Sums of Geometric Series

When a_1 is the first term, r is the common ratio, $r \neq 1$, and n is the number of terms.	
Formula:	
Sum of a Finite Geometric Sequence with ratio $r \neq 1$	$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$
Alternate Form of the Finite Formula	$S_n = \frac{a_1 - a_1 \cdot r^n}{1-r}$
Sum of an Infinite Geometric Series if $\lvert r \rvert < 1$	$S = \frac{a_1}{1-r}$

NOTE: An infinite geometric series with $|r| \geq 1$ does not have a finite sum. The series is said to **diverge.**

EX #1: What do you need to find the sum of the first 8 terms in the series $3+6+12+24+\ldots$?

a_1 = first

r = ratio

n = number of terms

$S_8 = 3\left[\frac{1-2^8}{1-2}\right] \Rightarrow 765$

EX #2: Find the sum of the finite geometric series

A. $3+6+12+24+\ldots+1536$

$a_1 = 3$
$r = 2$
$n = ? = 10$

1) Find n for 1536

$1536 = 3(2)^{n-1}$
$512 = 2^{n-1}$

$\log_2 512 = n-1$
$9 = n-1$
$10 = n$

2) $S_{10} = \frac{a_1(1-r^n)}{1-r}$

$S_{10} = \frac{3(1-2^{10})}{1-2}$

$S_{10} = 3069$

B. $-15+30-60+120-240+480$

$a_1 = -15$
$r = -2$
$n = 6$

$S_6 = -15\left[\frac{1-(-2)^6}{1-(-2)}\right]$

$S_6 = -15\left[\frac{1-64}{3}\right]$

$S_6 = (-5)(-63)$

$S_6 = 315$

EX #3: Find the sum of the finite geometric series.

A. $\sum_{n=1}^{10} 5(-2)^{n-1}$

$a_1 = 5$
$r = -2$
$n = 10$

$S_{10} = 5\left[\frac{1-(-2)^{10}}{1-(-2)}\right]$

$= 5\left[\frac{-1023}{3}\right]$

$= -1705$

B. $\sum_{k=0}^{24} 3\left(\frac{1}{2}\right)^n$ (* at k=0)

* Caution:
$24 - 0 + 1 = 25$ terms

$a_1 = 3$
$r = \frac{1}{2}$
$n = 25$

$S_{25} = 3\left[\frac{1-(\frac{1}{2})^{25}}{1-\frac{1}{2}}\right]$

$S_{25} = 3\left[\frac{0.9999999702}{.5}\right]$

$S_{25} \approx 6$

Using the Geometric Series Formula

EX #5: You make a contract with your parents to work eight hours every day during the month of July. The first day you will be paid one penny, the second day two cents, on the third day you get 4 cents, the fourth day 8 cents, and so on. Is this a good plan for you or do your parents get the most out of the contract?

July 31 days

$a_1 = 0$

$n = 30$

S_{30}

1¢, 2¢, 4¢, 8¢, ... ?

$2^0, 2^1, 2^2, 2^3, \ldots \; 2^{30}$

$$S_{30} = 1\left[\frac{1-2^{30}}{1-2}\right]$$

$$= 1\left[\frac{-1073741823}{-1}\right]$$

$= 1073741823$ pennies

How many dollars? $10,737,418.23

Wow, that's a deal!

Series and Summation

EX#6: Write the summation notation and find the sum of the first 6 terms of the geometric series $3 + 15 + 75 + 375 + \ldots$

1) formula $a_n = 3(5)^{n-1}$

$a_1 = 3$

$r = 5$

$n = 6$

2) summation:

$$\sum_{n=1}^{6} \left[3(5)^{n-1}\right]$$

3) Sum:

$$S_6 = \frac{3(1-5^6)}{1-5} \Rightarrow \frac{3(-15624)}{-4} \Rightarrow 11,718$$

EX #7: Write the summation notation and find the sum of the first 10 terms of the geometric series $2-1+\frac{1}{2}-\frac{1}{4}+\ldots$

$a_1 = 2$
$r = -\frac{1}{2}$
$n = 10$

1) rule: $a_n = 2\left(-\frac{1}{2}\right)^{n-1}$

2) Summation: $\sum_{n=1}^{10}\left[2\left(-\frac{1}{2}\right)^{n-1}\right]$

3) Sum: $S_{10} = \dfrac{2\left(1-\left(-\frac{1}{2}\right)^{10}\right)}{1-\left(-\frac{1}{2}\right)}$

$S_{10} = 2\left[\dfrac{1-\frac{1}{1024}}{\frac{3}{2}}\right] \Rightarrow 2\left[\dfrac{\frac{1024-1}{1024}}{\frac{3}{2}}\right]$

$\Rightarrow 2\left[\frac{1023}{1024}\cdot\frac{2}{3}\right] \Rightarrow \frac{341}{256}$ (with 1023 → 341 and 1024 → 512 crossed out)

Analyzing Infinite Geometric Series

EX #8: Determine whether the series *converges* or *diverges*. If it converges, find the sum.

A. $\frac{1}{2}+\frac{3}{4}+\frac{9}{8}+\ldots$

$r = \frac{3}{4} \div \frac{1}{2}$
$r = \frac{3}{2}$

$r > 1$ the series diverges

B. $\frac{1}{3}-\frac{1}{9}+\frac{1}{27}-\frac{1}{81}+\ldots$

$|r|<1$ the series converges

$r = -\frac{1}{9} \div \frac{1}{3}$

$r = -\frac{1}{3}$

$S = \dfrac{-\frac{1}{3}}{1-(-\frac{1}{3})}$

$= -\frac{1}{3} \cdot \frac{3}{4}$

$S = -\frac{1}{4}$

C. If the series converges can you use a calculator to find the sum?

$\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$

$S = \dfrac{\frac{2}{3}}{1-\frac{2}{3}}$

$S = \dfrac{\frac{2}{3}}{\frac{1}{3}}$ $S=2$

On a calculator, you can use 50 for ∞
Sum is 1.999
Use 100 for ∞
Sum is 2.

Name March 16 Date #1-12 Period

Arithmetic Sequences Homework

Problems 1 – 4, is the given sequence arithmetic? If so, identify the common difference.

1. $1, 1, 2, 3, 5, 8, \ldots$	2. $-83, -74, -65, -56, \ldots$
3. $4, \frac{5}{2}, 1, \frac{-1}{2}, \ldots$	4. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

Problems 5 – 8, find the 28th term of each sequence.

5. $22, 25, 28, 31, \ldots$	6. $5, 3, 1, -1, \ldots$
7. $1.7, 1.9, 2.1, 2.3, \ldots$	8. $23, 16, 9, 2, \ldots$

Problems 9 – 12, find he missing term of each arithmetic sequence.

9. $15, \square, 41, \ldots$	10. $\ldots, 7, \square, 24, \ldots$
11. $\frac{9}{2}, \square, \frac{47}{2}$	12. $\ldots, -3, \square, 437, \ldots$

March 17 #13-24

Problems 13 – 16, find the arithmetic mean a_n of the given terms.

13. $a_{n-1} = 12, a_{n+1} = 2$	14. $a_{n-1} = 0.3, a_{n+1} = 2.9$
15. $a_{n-1} = m, a_{n+1} = m + n$	16. $a_{n-1} = 5x + 9, a_{n+1} = x + 7$

Problems 17 – 20, find the 14th term of each sequence.

17. $a_{13} = 15, d = 4$	18. $a_{13} = 15, d = -7$
19. $a_{15} = 15, d = -11$	20. $a_{15} = 15, d = 19$

Problems 21 – 24, write an explicit and a recursive formula for each sequence.

21. $0, 3, 6, 9, 12, \ldots$	22. $13, 8, 3, -2, -7, \ldots$
23. $-7, 4, 15, 26, 37, \ldots$	24. $-28, -16, -4, 8, 20, \ldots$

March 18 #25-29

Problems 25 – 26, find the missing terms of each arithmetic sequence. [*Hint:* The arithmetic mean of the first and fifth terms is the third term.]

25. 3, $\square$, $\square$, $\square$, – 25,...

26. $\frac{14}{3}$, $\square$, $\square$, $\square$, $\frac{22}{3}$,...

Problems 27 – 29, solve.

27. You know your school bus stops at 2 minute intervals. The first stop of the route begins at 6:20 A.M. every day. If your stop is the 18th stop, when will the bus arrive for you?

28. Your grandparents gave you a $500 gift during the holidays. You plan to add $25 to the gift every week to save for summer break, which begins in June.

A. Write the amount in the account after each deposit as an arithmetic sequence.

B. If there are 23 weeks before your vacation begins, How much money will you have saved by summer?

29. Each school year, Mu Alpha Theta plans to add 5 more members to their tutoring team for helping math students who struggle. This year there were 37 student tutors.

A. Write a recursive formula for the number of tutors the club expects to add each year.

B. Write the first five terms of the sequence.

C. How many tutors will the club have in the 15th year if this rate of increase remains steady?

Name March 19 #1–12 Date ____________ Period ______

Geometric Sequences Homework

Problems 1 – 4, is the given sequence geometric? If so, identify the common ratio and find the next two terms.

1. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$	2. $18, -6, 2, \frac{-2}{3}, \ldots$
3. $9, 0.9, 0.09, 0.009, \ldots$	4. $0.5, 0.25, 0.125, 0.0625, \ldots$

Problems 5 – 8, write the explicit formula for each sequence. Then generate the first five terms.

5. $a_1 = 8, r = -3$	6. $a_1 = 0.0175, r = 5$
7. $a_1 = 13, r = -\frac{1}{2}$	8. $a_1 = 6561, r = \frac{1}{3}$

Problems 9 – 12, find the missing term of each geometric sequence.

9. $637, \square, 13, \ldots$	10. $\ldots, 8, \square, 1458, \ldots$
11. $12, \square, 17.28$	12. $\ldots, \frac{3}{5}, \square, \frac{27}{80}, \ldots$

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March 20 #13 – 24

Problems 13 – 16, identify each sequence as *arithmetic, geometric,* or *neither.* Then find the next two terms.

13. $1, -2, 4, -8, \ldots$	14. $40, 45, 50, 55, \ldots$
15. $-4, 12, -36, 108, \ldots$	16. $11, 12, 14, 17, 22, \ldots$

Problems 17 – 20, find the missing terms of each geometric sequence.

17. $2187, \square, \square, \square, 27, \ldots$	18. $12, \square, \square, \square, 60.75, \ldots$
19. $0.1024, \square, \square, \square, 4, \ldots$	20. $\frac{1}{2}, \square, \square, \square, \frac{8}{81}, \ldots$

Problems 21 – 24, for the geometric sequence $2, 8, 32, 128, \ldots$ find the indicated term.

21. 5^{th} term	22. 13^{th} term
23. 8^{th} term	24. nth term

March 23 #25-28

Problems 25 – 26, write the explicit formula for the sequence. Then generate the first five terms.

25. $a_1 = 5, r = -2$	26. $a_1 = 729, r = \frac{-1}{3}$

Problems 27 – 28, solve.

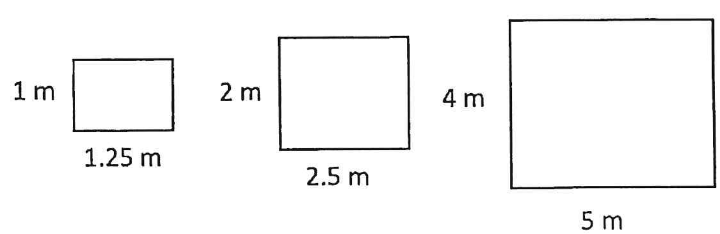
27. Jordan got some helium balloons from her best friend Madison on her birthday. The balloons lose one-fifth of the helium each day. Each balloon was filled to a volume of 2500 in^3.

A. Write the geometric sequence representing the amount of helium in the balloons for the first five days.

B. What is the common ratio of the sequence? [*Hint*: Volume is in cubic inches.]

C. How much helium will be in the balloons on the 10^{th} day?

28. Find the perimeter, in meters, of the sixth rectangle in the pattern.



 Jean Adams

Name March 26 #1-8 Date _______ Period _______

Arithmetic Series Homework

Problems 1 – 4, for each sum, find the number of terms, the first term, and the last term. Then evaluate the series.

1. $\sum_{n=1}^{5}(n+2)$	2. $\sum_{n=2}^{6}(2n-1)$
3. $\sum_{n=3}^{6}(3n+2)$	4. $\sum_{n=4}^{10}(n)$

Problems 5 – 8, write the related series for each finite sequence. Then evaluate each series.

5. $4, 5, 6, 7, \ldots 12$	6. $4, 7, 10, \ldots 31$
7. $1, 6, 11, 16, \ldots 46$	8. $3, 6, 9, 12, \ldots 42$

Problems 9 – 12, use summation notation to write each arithmetic series for the specified number of terms.

March 27 #9-16

9. $1+3+5+\ldots; n=7$	10. $4.3+4.6+4.9+\ldots; n=5$
11. $10+7+4+\ldots; n=8$	12. $(-3)+(-6)+(-9)+\ldots; n=6$

Problems 13 – 16, tell whether each list is a *series* or a *sequence*. Then tell whether it is *finite* or *infinite.*

13. 5, 9, 13, 17, 21	14. $-\frac{1}{2}-\frac{1}{4}-\frac{1}{8}-\frac{1}{16}-\ldots$
15. 1.8+3.6+7.2+14.4	16. 1,2,4,8,16,32,...

Problems 17 – 18, solve March 30 #17-18

17. Isabella scored 76% on her first math test this quarter. Her next two scores were 80% and 84%. If she continues to improve at this rate,

A. What would be her score on the fifth (and final) test for the quarter?

B. Assuming each test is worth 100 points, what is Isabella's percentage for tests?

18. The school auditorium has 20 rows and two aisles, The two side sections have 6 seats in the first row and one more seat in each succeeding row. The middle section has 10 seats in the first row and one additional seat in each succeeding row.

A. Find the total seats in each section. Then find the total seating capacity of the auditorium.

B. Write an arithmetic series to represent each of the sections.

C. The drama club tickets for the spring play are $15 for the first five rows, and $10 for rows 6 – 15. The last five rows sell for $7 each. What is the total amount of money that could be generated by a full house.

Name March 31 #1-8 Date ____________ Period ________

Geometric Series Homework

Problems 1 – 4, evaluate the series to the given term.

1. $2+4+5+\ldots; S_7$	2. $-3-6-12-\ldots; S_7$
3. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots; S_6$	4. $-\frac{1}{5}+1-5+25\ldots; S_5$

Problems 5 – 8, decide whether each infinite geometric series *diverges* or *converges*. State whether each series has a sum.

5. $1+\frac{1}{2}+\frac{1}{4}+\ldots$	6. $3-3+3-\ldots$
7. $1-\frac{1}{4}+\frac{1}{16}-\ldots$	8. $\frac{1}{4}+\frac{1}{2}+1+2+\ldots$

Problems 9 – 12, evaluate each infinite series that has a sum April 1 #9-16

9. $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^{n-1}$	10. $\sum_{n=1}^{\infty}4\left(2^{n-1}\right)$

11. $\sum_{n=1}^{\infty} 7\left(\frac{1}{4}\right)^{n-1}$	12. $\sum_{n=1}^{\infty} 5(1.5)^{n-1}$

Problems 13 – 16, determine whether each series is *arithmetic* or *geometric*. Then evaluate the series to the given term.

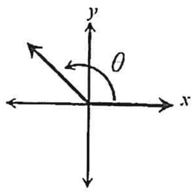
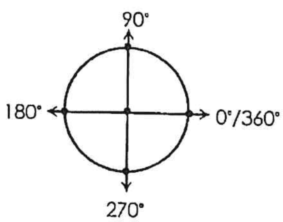
13. $3 + 6 + 9 + 12 + \ldots; S_{20}$	14. $3 + 6 + 12 + 24 + \ldots; S_{10}$
15. $-2 + 2 + 6 + 10 + \ldots; S_{12}$	16. $-4 + 16 - 64 + 256 - \ldots; S_9$

Problems 17 – 20, evaluate each infinite geometric series. April 2 #17-20

17. $1.1 + 0.11 + 0.011 + \ldots$	18. $4 + 1 + \frac{1}{4} + \frac{1}{16} + \ldots$
19. $1.1 - 0.11 + 0.011 - \ldots$	20. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

NTI 11 Pre-Cal Mills

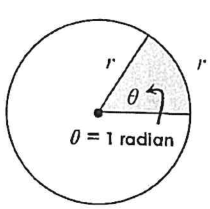
Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
Angles in **STANDARD FORM** 	• An angle on the coordinate plane is in standard form when the vertex is on the origin and one ray lies on the positive x-axis. • The ray on the x-axis is called the _____________ ________. • The other ray is called the _____________ ________. • Counterclockwise rotations result in _____________ angle measures. • Clockwise rotations result in _____________ angle measures.
DEGREES 	• The most common unit of measure for angles is the ___________ (°). • A degree is equivalent to ___________ of a full rotation.

Sketch an angle in standard form with the given measure.

1. 65°	2. 320°	3. 415°
4. -140°	5. -15°	6. -210°

RADIANS



A **radian** is a unit of angle measure based on arc length. One radian is defined as the measure of the angle formed when the radius is equivalent to the length of the intercepted arc. Recall that the circumference of a circle is $2\pi r$, therefore:

360° = ____________________; **180°** = ____________________

Converting Degrees → Radians	Converting Radians → Degrees
$\text{Radians} = \text{Degrees} \cdot \left(\frac{\pi \text{ radians}}{180}\right)$	$\text{Degrees} = \text{Radians} \cdot \left(\frac{180}{\pi \text{ radians}}\right)$

Converting DEGREES TO RADIANS

Convert each angle measure to radians.

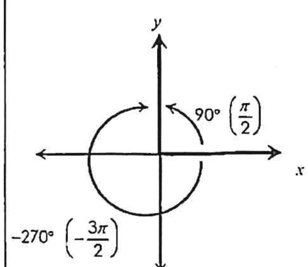
7. 60°	8. 160°	9. 330°
10. -45°	11. -230°	12. -75°

Converting RADIANS TO DEGREES

Convert each angle measure to degrees.

13. $\frac{5\pi}{18}$	14. $\frac{11\pi}{9}$	15. $\frac{7\pi}{3}$
16. $-\frac{\pi}{6}$	17. $-\frac{19\pi}{12}$	18. $-\frac{3\pi}{4}$

COTERMINAL ANGLES



Angles in standard position with the same terminal side are coterminal angles. Determine whether the two angles are coterminal.

19. 305° and -55°	20. 30° and -170°
21. $\frac{\pi}{4}$ and $-\frac{9\pi}{4}$	22. $\frac{3\pi}{2}$ and $-\frac{5\pi}{2}$

Give two coterminal angles for each given angle, one positive and one negative.

23. 150°	24. 265°	25. -25°
26. $\frac{\pi}{12}$	27. $\frac{7\pi}{6}$	28. $-\frac{11\pi}{18}$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
DEGREE-MINUTE-SECOND FORM (DMS)	Some angles have decimal degrees (for example, 26.5°). Angles with decimals can be expressed using **degree-minute-second form (DMS)** in which the degrees are subdivided into minutes and seconds as follows:

one degree = __________ minutes; one minute = __________ seconds

***one degree = ______________ seconds**

Symbols for degree, minutes, and seconds:

DEGREE	MINUTE	SECOND

Converting DECIMAL DEGREES TO DMS

Follow the example below to convert 31.875° to degree-minute-second form.

31.875° ➡ [ ] · [ ] = [ ]

[ ] · [ ] = [ ]

Therefore, 31.875° = [ ]

Write each angle measure in DMS form.

1. 15.48°	**2.** 150.2°
3. 89.915°	**4.** 340.375°
5. 48.526°	**6.** 273.771°

	7. 105.394°	**8.** 318.812°

Converting DMS TO DECIMAL DEGREES

Follow the example below to convert 73° 9' 50" to decimal degrees.

73° 9' 50" ➡ [ ] / [ ] = [ ]

[ ] / [ ] = [ ]

Therefore, 73° 9' 50" = [ ] = [ ]

Write each angle measure in decimal degree form.

9. 37° 36'	**10.** 315° 8'
11. 84° 52' 18"	**12.** 121° 4' 45"
13. 310° 28' 20"	**14.** 8° 34' 16"
15. 50° 14' 56"	**16.** 253° 46' 5"

Name: ______________________________ **Unit 5:** Trigonometric Functions

Date: ____________________ Per: ______ **Homework 1:** Angle Measures

**** This is a 2-page document! ****

Directions: Convert each angle measure to radians.

1. 140°	**2.** -75°	**3.** 190°
4. 312°	**5.** -65°	**6.** -225°

Directions: Convert each angle measure to degrees.

7. $\frac{3p}{5}$	**8.** $-\frac{7p}{18}$	**9.** $\frac{10p}{9}$
10. $-\frac{9p}{5}$	**11.** $-\frac{11p}{12}$	**12.** $\frac{15p}{4}$

Directions: Give two coterminal angles for each given angle, one positive and one negative.

13. 195°	**14.** 319°	**15.** -64°
16. $\frac{3p}{5}$	**17.** $\frac{11p}{7}$	**18.** $\frac{19p}{6}$

Directions: Write each angle measure in Degree-Minute-Second (DMS) form.	
19. 36.875°	**20.** 117.590°
21. -238.145°	**22.** 64.901°
23. -323.588°	**24.** 9.636°

Directions: Write each angle measure in decimal degree form.	
25. 54° 26' 18"	**26.** 184° 37' 20"
27. -98° 12' 48"	**28.** 289° 5' 30"
29. -303° 20' 14"	**30.** 100° 45' 6"

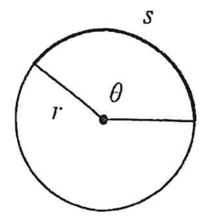
Name:

Date:

Topic:

Class:

Main Ideas/Questions	Notes/Examples
ARC LENGTH	If θ is a central angle in a circle with radius r, then the length of the intercepted arc, s, is given by: [ ] (where θ is measured in radians)



Examples

Find the length of the intercepted arc given the angle measure and radius. Give your answer in terms of π and rounded to the nearest tenth.

1. $\frac{5\pi}{6}$; $r = 3$ ft

2. $\frac{\pi}{4}$; $r = 14.5$ in

3. $\frac{3\pi}{2}$; $r = 0.5$ yd

4. 2π; $r = 6.2$ yd

Convert to radians!

5. $15°$; $r = 8.4$ m

6. $330°$; $r = 10.5$ mm

7. $120°$; $r = 1.8$ ft

8. $135°$; $r = 25$ km

9. The central angle θ in a circle of radius 6 meters has an intercepted arc length of 10 meters. Find the measure of θ in radians and in degrees.

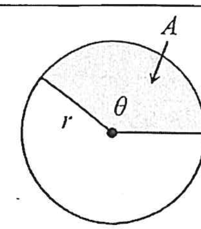
10. A merry go round rotates 2808° per ride. How far would a rider seated 8 feet from the center of the merry go round travel during the ride?

11. Cincinnati, Ohio is directly north of Atlanta, Georgia. Cincinnati has a latitude of 39.1° N and Atlanta has a latitude of 33.7° N. If the earth has a radius of 3,960 miles, how far apart are these cities?

AREA OF SECTORS

The area A of a sector of a circle with radius r and central angle θ is given by:

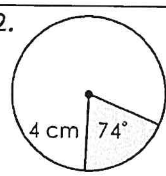
(where θ is measured in radians)



Examples

Find the area of each sector.

12.



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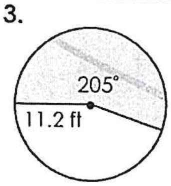

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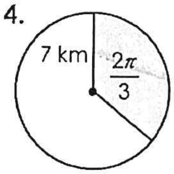

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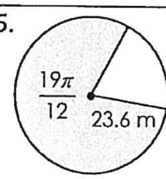
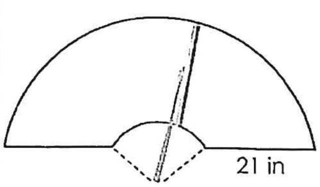


16. The area of a sector of a circle with a central angle of 240° is 134 ft^2. Find the radius of the circle.



17. The windshield wiper arm to the left is 32 inches long. If the wiper sweeps through an angle of 125°, find the area swept by the blade.

Name: ____________________ Unit 5: Trigonometric Functions

Date: ____________ Per: ______ Homework 2: Arc Lengths & Area of Sectors

** This is a 2-page document! **

Directions: Find the length of each intercepted arc given the angle measure and radius. Give your answer in terms of π and rounded to the nearest tenth.

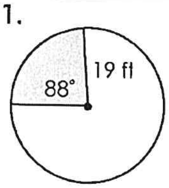
1. $\frac{3\pi}{4}$; r = 10 mm	2. $\frac{8\pi}{7}$; r = 17.5 yd
3. $\frac{\pi}{6}$; r = 2 mi	4. $\frac{5\pi}{8}$; r = 18 ft
5. 320°; r = 0.9 cm	6. 100°; r = 4.5 yd
7. 240°; r = 6.6 km	8. 45°; r = 2.2 m

9. An intercepted arc has a length of 19 yards. If the radius is 4 yards, find the measure of the central angle in radians and degrees.

10. The end of minute hand on a clock is 5.5 inches from the center. How far will the minute hand travel over three full days?

Directions: Find the area of each sector.

11.



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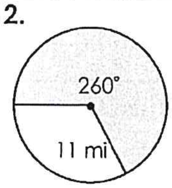

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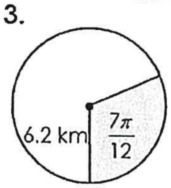

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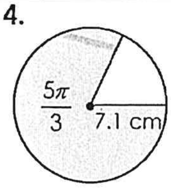

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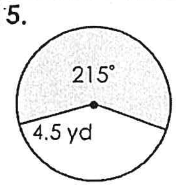

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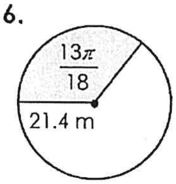


17. The sector of a circle has a central angle of 144° and an area of 70.7 square meters. Find the radius of the circle.

18. A circle with a radius of 15 yards contains a sector with an area of 609 yd^2. Find the measure of the central angle of the sector in both radians and degrees.

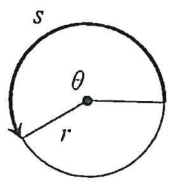
19. A wall clock is equally divided into 12 sections. If the clock reads 8:00 and has a diameter of 12.5 inches, find the area of the smaller sector formed by the minute and hour hands.

Name:

Date:

Topic:

Class:

Main Ideas/Questions	Notes/Examples	
CIRCULAR MOTION 	**Arc length can be used to analyze circular motion. Suppose an object moves along a circular path s with radius r:**	
	The rate at which the object moves along the path is called its **linear speed, v.**	The rate at which the central angle changing along the path is called its **angular speed, ω.**
	Linear Speed Formula:	**Angular Speed Formula:**
Examples	**Use for questions 1-2:** A bicycle tire with a radius of 14 inches rotates at a rate of 125 revolutions per minute (rpm):	
Recall: Each rotation (or revolution) has a circumference of $2\pi r$ and a central angle of 2π radians.	**1.** Find the linear speed of the tire in inches per minute.	**2.** Find the angular speed of the tire in radians per minute.
	Use for questions 3-4: A CD with a diameter of 120 millimeters rotates a rate of 45 revolutions per minute.	
	3. Find the linear speed of the CD in millimeters per minute.	**4.** Find the angular speed of the CD in radians per minute.
	5. A 16-inch diameter tire on a car is making 500 revolutions per minute. Find the linear speed of the tire in miles per hour.	

6. A circular saw blade with a diameter of 9 inches rotates at 2800 revolutions per minute. Find the angular speed of the blade in radians per second.

7. A windmill has blades that are 14 feet long. If the windmill is rotating at 5 revolutions per second, find the linear speed of the tips of the blades in miles per hour.

Using ANGULAR SPEED to find LINEAR SPEED

The linear speed, v, can also be found as follows:

v = ☐ = ☐ = ☐ = ☐

Therefore, you can use the angular speed, ω, to find the linear speed, v.

8. A ceiling fan with 25-inch blades rotates at 40 rpm. Find the linear speed of the tips of the blades in feet per second.

9. Ryan is riding a bicycle whose wheels are 28 inches in diameter. If the wheels rotate at 130 rpm, find the linear speed in miles per hour in which he is traveling.

QUIZIZZ

NAME : CCR

CLASS :

DATE :

Solving Linear Equations

13 Questions

1. Solve for x:
   2x + 5 = 12

a) 7
b) 3
c) 3.5
d) 8.5

2. Solve for x:
   4x + 1 = 7x - 5

a) -1
b) 2
c) 6/11
d) 8

3. Solve for x:
   2 - 4x = 3x - 12

a) 2
b) -14
c) 10
d) -2

4. Solve for x:
   6(2x + 1) = 18

a) 3
b) 17/12
c) 1
d) 2

5. Solve for x:
   2(3x + 4) + 2 = 4 + 3x

a) 2
b) -2/3
c) 10
d) -2

6. Solve for x:
   10 = 3x -12

a) 22/3
b) 2/3
c) 10/9
d) 8

7. Solve for x:
$7x + 5 + x - 3 + x = 5$

a) 1/3
b) 3
c) 3/7
d) 1

8. Solve for x:
$7 - 5x = 12$

a) 1
b) -6
c) -1
d) 6

9. Solve for x:
$7 - 2(10x + 1) = 25$

a) 2/5
b) 2
c) 1
d) -1

10. Solve for x:
$8x + 2 + 3x + 5 = x + 12$

a) -1/2
b) 2
c) -2
d) 1/2

11. Solve for x:
$12 = 20x - 18$

a) 1
b) -2
c) -1
d) 2

12. Solve for x:
$5 + 3x = 8x - 15$

a) 10
b) 4
c) 2
d) 20/11

13. Solve for x:
$3(5x + 10) + 10 = 6x - (x + 10)$

a) 3
b) -3
c) -5
d) 5

QUIZIZZ

NTI 12

NAME : ____________________

CLASS : ____________________

DATE : ____________________

Solving Linear Equations

20 Questions

1. **Solve:**

$$3x + 1 = 13$$

a) 2
b) 4
c) 6
d) 8

2. **Solve:**

$$3x + 1 = 13$$

a) 1
b) 2
c) 3
d) 4

3. **Solve:**

$$5x - 3 = 27$$

a) 5
b) 6
c) 7
d) 8

4. Solve:

$$8x + 4 = 36$$

a) 2
b) 3
c) 4
d) 5

5. Solve:

$$2 = \frac{x}{5} - 4$$

a) 32
b) 30
c) 29
d) 20

6. Solve:

$$2m - 31 = 17$$

a) 24
b) 25
c) 26
d) 28

7. Solve:

$$\frac{x}{9} - 3 = 2$$

a) 35
b) 40
c) 42
d) 45

8. **Solve:**

$$3(2x - 1) = 21$$

a) 1
b) 2
c) 3
d) 4

9. **Solve:**

$$-2(4x + 6) = 28$$

a) 5
b) -5
c) 7
d) -7

10. **Solve:**

$$7(2 - 3x) = 56$$

a) 2
b) -1
c) -2
d) 1

11. **Solve:**

$$8(x + 1) = 2(x - 2)$$

a) 2
b) 1
c) -1
d) -2

12. Solve:

$$7(2x+4) = 10(x+4)$$

a) 1
b) 2
c) 3
d) 4

13. Solve:

$$3x - 1 = 2x + 6$$

a) 7
b) 8
c) 9
d) 10

14. Solve:

$$5x - 10 = 3x + 6$$

a) 6
b) 8
c) 2
d) 4

15. Solve:

$$\frac{3x-7}{5} = 4$$

a) 7
b) 8
c) 9
d) 10

16. **Solve:**

$$11 = \frac{5x - 2}{3}$$

a) 7
b) 6
c) 5
d) 8

17. **Solve:**

$$3(x - 2) = 2x - 1$$

a) 4
b) 5
c) 6
d) 10

18. **Solve:**

$$3(2x - 5) - 5(x - 4) = 0$$

a) -5
b) -2
c) 2
d) 5

19. **Solve:**

$$7x - 8 = 2x + 7$$

a) 1
b) 2
c) 3
d) 4

20.

Solve:

$$4(2x-2)+3(x-4)=57$$

a) 3

b) 5

c) 7

d) 9

QUIZIZZ

NAME : ____________

CLASS : ____________

DATE : ____________

Solving Linear Equations

NTI 13

15 Questions

1. In the expression 3m + 8, the number 3 is called the ____________.

a) variable
b) constant
c) coefficient
d) answer

2. The first step to solving the equation g - 3 = -12 is to . . .

a) divide by -3 on both sides of the equal sign
b) multiply by -3 on both sides of the equal sign
c) subtract 3 on both sides of the equal sign
d) add 3 on both sides of the equal sign

3. From the last equation g - 3 = -12, after you add 3 on both sides of the equal sign, what is your answer?

a) g = -15
b) g = -9
c) g = 36
d) g = 15

4. In the expression 3m + 8, the number 8 is called the ____________.

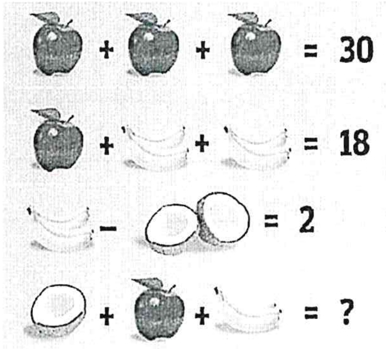
a) constant
b) variable
c) coefficient
d) answer

5. What is the first step in solving for w in the equation 3w + 5 = 14?

a) Add 5 to both sides of the equal sign
b) Divide by 3 on both sides of the equal sign
c) Subtract 5 from both sides of the equal sign
d) Multiply by 3 on both sides of the equal sign

6. Solve the equation 4n - 7 = -23

a) n = -4
b) n = 4
c) n = 7.5
d) n = -7.5

7. 

Solve.

a) 12
b) 13
c) 14
d) 15

8. Distribute 5(b - 2) and you get . . .

a) 5b - 2
b) 5b - 10
c) 5b + 10
d) 3b

9. Solve the equation 5(b - 2) = -10

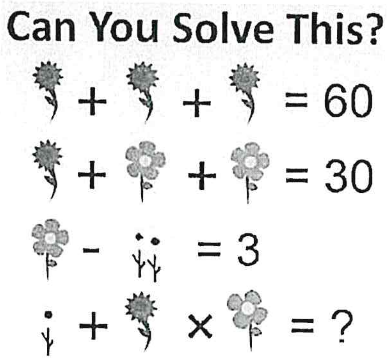
a) b = 4
b) infinite solutions
c) no solution
d) b = 0

10. In the equation 5d - 1= 2d - 7, we need to get the variable to one side of the equal sign. What would we do to move the variable to the left side?

a) -2 from both sides of the equal sign
b) -5d from both sides of the equal sign
c) -2d from both sides of the equal sign
d) + 7 on both sides of the equal sign

11. In the equation 5d - 1= 2d - 7, after we -2d from both sides, we get 3d - 1 = -7. What does d = if you finish solving it?

a) d = -1
b) d = 1
c) d = 2
d) d = -2

12. 

Solve.

a) 81
b) 101
c) 110
d) 84

13. Which equation below would give you the answer no solution?

a) 3x + 3 = 3x + 3
b) 3x + 3 = 6x + 6
c) 3x + 3 = 3x + 2
d) 3x + 3 = 0

14. Which equation below would give you an answer of infinite solutions (all real numbers)?

a) 4x + 10 = 0
b) 4x + 10 = 4x
c) 4x + 10 = 4x + 8
d) 4x + 10 = 4x + 10

15. In the equation 4c + 8 = 14, c is the ______________.

a) constant
b) coefficient
c) calculation
d) varialbe

QUIZIZZ

Solving Inequalities

10 Questions

NAME : ______________________

CLASS : ______________________

DATE : ______________________

1. Solve the inequality.
   $x + 5 \leq 13$

a) $x \geq 8$
b) $x \geq 18$
c) $x \leq 8$
d) $x \leq 18$

2. $4h<8$

a) $h<32$
b) $h<2$
c) $h<4$
d) $h>2$

3. Which is the solution of
   $-4 - 6m > -9m + 8$?

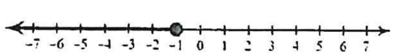
a) $m < -4$
b) $m > 4$
c) $m < 15$
d) $m > 36$

4. Solve the inequality for w:
   $-5(w - 4) < -5w - 15$

a) $w > 15$
b) All real numbers are solutions
c) $w < 5$
d) No solutions to this inequality

5. Four times a number, increased by 6 is greater than 22.
   Which inequality models this scenario?

a) $4x - 6 > 22$
b) $4x + 6 < 22$
c) $4x + 6 \geq 22$
d) $4x + 6 > 22$

6. 

A) $m \leq -1$ B) $m > 1$
C) $m \geq -1$ D) $m < -1$

Which inequality matches the graph?

a) A
b) B
c) C
d) D

7. When graphing an inequality, what does a closed circle mean?

a) The number is not included in the list of answers.

b) The number is included in the list of answers.

8. Which numbers are possible solutions to the inequality...
(SELECT ALL THAT APPLY)-5x + 10 < 30

a) 0

b) 100

c) -100

d) -3

e) -4

9. Which numbers are possible solutions to the inequality...
(SELECT ALL THAT APPLY)-5x + 10 ≥ 30

a) 0

b) 100

c) -100

d) -4

e) -5

10. Which numbers are possible solutions to the inequality...
(SELECT ALL THAT APPLY)x+6x≤2(2+3x)

a) 4

b) 100

c) 5

d) 0

e) -100

QUIZIZZ

NAME : ____________________

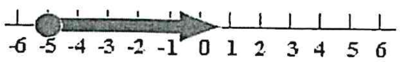
CLASS : ____________________

DATE : ____________________

Solving Inequalities

NTI 15

13 Questions

1. 

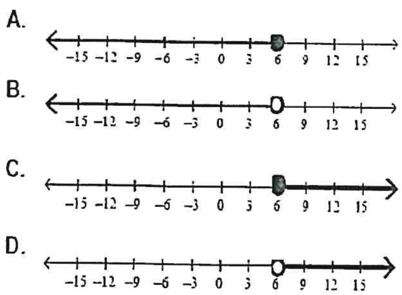
What inequality does the number line graph represent?

a) $x > 5$

b) $x < -5$

c) $x \geq 5$

d) $x \geq -5$

2. 

Pick the correct letter for:
$6 > x$

a) A

b) B

c) C

d) D

3. $10 \geq x - 3$

a) $x \leq 13$

b) $x \geq 13$

c) $x \leq 7$

d) $x \geq 7$

4. $-4x + 14 \leq 54$

a) $x \geq -10$

b) $x \leq -10$

c) $x \geq 10$

d) $x \leq 10$

5. $5z - 2 - 2z < 13$

a) $z > 5$

b) $z < 5$

c) $z > 1$

d) $z < 1$

6. Does the sign need to flip when solving?
$8y < (-40)$

a) YES

b) NO

7. Do you flip the sign?
$-3x + 2 \le 22$

a) YES

b) NO

8. If 15 > x - 1 then a possible set of solutions for x is...

a) 1, 7, 15

b) 19, 22, 30

c) 80, 100

9. Solve:
$3(x - 2) < 2(x + 9)$

a) x<24

b) x>12

c) x<12

d) x>24

10. Solve:
$7(x + 3) < 5x + 13$

a) x>-4

b) x < 17

c) x<-4

d) x > -17

11. You flip an inequality symbol when you...

a) subtract

b) multiply and divide only

c) multiple by a negative number

d) multiple or divide by a negative number

12. Your family needs 2 gallons of water this week, but you don't want to buy any more than 8 gallons. Write an inequality representing the possible gallons of water you could buy this week.

a) $2 \le x \le 8$

b) $2 < x < 8$

c) $8 \ge x \ge 2$

d) $2 > x > 8$

13. At the local 5k race, runners 10 and under or runners 65 and above start the race after the others. What compound inequality represents the situation?

a) $10 \le x \le 65$

b) $x \ge 10$ or $x \le 65$

c) $x \le 10$ or $x \ge 65$

d) $10 < x < 65$

Quizizz

NAME : ______________________

CLASS : ______________________

DATE : ______________________

NTI 16

Solving Inequalities

14 Questions

1. Does the sign need to flip when solving?
   -2x ≥ 22

a) YES

b) NO

2. Solve
   -5m < 10

a) m < -2

b) m > -2

c) m < -50

d) m > -50

3. Solve the inequality.
   x + 5 ≤ 13

a) x ≥ 8

b) x ≥ 18

c) x ≤ 8

d) x ≤ 18

4. 10 ≥ x - 3

a) x ≤ 13

b) x ≥ 13

c) x ≤ 7

d) x ≥ 7

5. -8x < 48

a) x < -6

b) x > -6

c) x < 6

d) x > 6

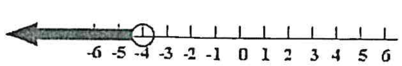
6. x - 4 > 5

a) x < 9

b) x ≥ 9

c) x > 1

d) x > 9

7. 

What inequality does the number line graph represent?

a) x ≤ 4

b) x ≥ -4

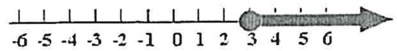
c) x < -4

d) x < 4

8. 

What inequality does the number line graph represent?

a) $-x + 4 \geq -1$
b) $-2x + 3 > -7$
c) $-x + 1 < -4$
d) $2x + 3 \geq 13$

9. 

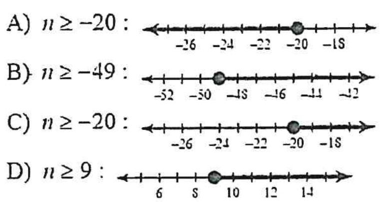
What inequality does the number line graph represent?

a) $x \geq 3$
b) $x > 3$
c) $x < -3$
d) $x \leq 3$

10. 

What inequality does the number line graph represent?

a) $-x + 5 \geq 9$
b) $2x - 3 > 5$
c) $-x + 3 \leq 7$
d) $-2x + 6 > -2$

11. 

Solve and graph the inequality.
$77 \leq 5 + 8n$

a) A
b) B
c) C
d) D

12. A taxi cab costs $1.75 and $0.75 for each additional mile. You have $20 to spend on you ride. Which inequality could be solved to find how many miles you can travel, if m is the number of additional miles?

a) $1.75n + 0.75 \geq 20$
b) $1.75n + 0.75 \leq 20$
c) $0.75n + 1.75 \geq 20$
d) $0.75n + 1.75 \leq 20$

13. The 30 members of a choir are trying to raise at least $1,500 to cover travel costs to a singing camp. They have already raised $600. Which inequality could you solve to find the average amounts each member can raise that will at least meet the goal?

a) $30x + 600 > 1,500$
b) $30x + 600 \geq 1,500$
c) $30x + 600 < 1,500$
d) $30x + 600 < 1,500$

14. Eleni bought 2 pounds of grapes at a cost of $3.49 per pound. She paid with a $10 bill. How much change did she get back?

a) $3.02

b) $4.51

c) $6.51

d) $6.98

QUIZIZZ

NAME : ______________

CLASS : ______________

DATE : ______________

Solving Linear Equations

15 Questions

NTI 17

1. In the expression 3m + 8, the number 3 is called the ___________.

a) variable
b) constant
c) coefficient
d) answer

2. The first step to solving the equation g - 3 = -12 is to . . .

a) divide by -3 on both sides of the equal sign
b) multiply by -3 on both sides of the equal sign
c) subtract 3 on both sides of the equal sign
d) add 3 on both sides of the equal sign

3. From the last equation g - 3 = -12, after you add 3 on both sides of the equal sign, what is your answer?

a) g = -15
b) g = -9
c) g = 36
d) g = 15

4. In the expression 3m + 8, the number 8 is called the ___________.

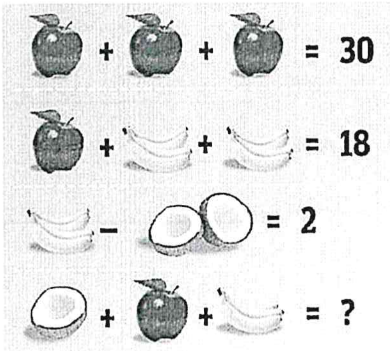
a) constant
b) variable
c) coefficient
d) answer

5. What is the first step in solving for w in the equation 3w + 5 = 14?

a) Add 5 to both sides of the equal sign
b) Divide by 3 on both sides of the equal sign
c) Subtract 5 from both sides of the equal sign
d) Multiply by 3 on both sides of the equal sign

6. Solve the equation 4n - 7 = -23

a) n = -4
b) n = 4
c) n = 7.5
d) n = -7.5

7. 

Solve.

a) 12

b) 13

c) 14

d) 15

8. Distribute 5(b - 2) and you get . . .

a) 5b - 2

b) 5b - 10

c) 5b + 10

d) 3b

9. Solve the equation 5(b - 2) = -10

a) b = 4

b) infinite solutions

c) no solution

d) b = 0

10. In the equation 5d - 1= 2d - 7, we need to get the variable to one side of the equal sign. What would we do to move the variable to the left side?

a) -2 from both sides of the equal sign

b) -5d from both sides of the equal sign

c) -2d from both sides of the equal sign

d) + 7 on both sides of the equal sign

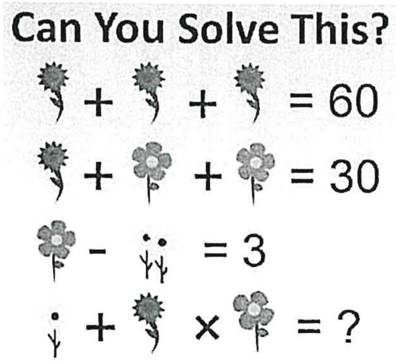
11. In the equation 5d - 1= 2d - 7, after we -2d from both sides, we get 3d - 1 = -7. What does d = if you finish solving it?

a) d = -1

b) d = 1

c) d = 2

d) d = -2

12. 

Solve.

a) 81

b) 101

c) 110

d) 84

13. Which equation below would give you the answer no solution?

a) $3x + 3 = 3x + 3$

b) $3x + 3 = 6x + 6$

c) $3x + 3 = 3x + 2$

d) $3x + 3 = 0$

14. Which equation below would give you an answer of infinite solutions (all real numbers)?

a) $4x + 10 = 0$

b) $4x + 10 = 4x$

c) $4x + 10 = 4x + 8$

d) $4x + 10 = 4x + 10$

15. In the equation $4c + 8 = 14$, c is the ______________.

a) constant

b) coefficient

c) calculation

d) varialbe

Quizizz

NAME : ____________

CLASS : ____________

DATE : ____________

Solving Linear Equations

NT I 18

12 Questions

1. Which operation will cancel out multiplication?

a) A. Addition
b) B. Subtraction
c) C. Multiplication
d) D. Division

2. Solve:
-4x = 24

a) 20
b) -20
c) 6
d) -6

3. Solve:
3x + 7 = -8

a) 5
b) -5
c) -12
d) -18

4. Solve:
7x - 3x - 8 = 24

a) 8
b) -8
c) 3.2
d) 4

5. Solve:
3(x - 2) = 18

a) 6.7
b) 4
c) 8
d) -4

6. After paying $8.24 for a sandwich,Amanda has $7.71. With how much money did she start?

a) $17.55
b) $24.19
c) $15.95
d) $0.50

7. Solve:
$7x + 19 = -2x + 55$

a) 4
b) 14.8
c) 8.2
d) -4

8. Solve:
$2(2x + 3) = -6(x + 9)$

a) 1.2
b) 6
c) -6
d) 7.5

9. What is the first step in solving this equation:
$5x + 7 = 12$

a) Divide by 5
b) Subtract 5
c) Multiply by 7
d) Subtract 7

10. $15 = \frac{x}{2}$ What step would you take to solve this equation?

a) Multiply both sides of the equation by 2
b) Divide both sides of the equation by 2
c) Add 2 to both sides of the equation
d) Subtract 2 from both sides of the equation

11. Find the error in the problem below
19 - 2k = 3k - 1
+3k +3k
19 + 1k = -1
-19 -19
k = -20

a) They should have divided by 1 for the last step
b) They should add 19 instead of subtracting 19
c) I can not find an error
d) Instead of adding 3k they should subtract 3k

12. $\frac{x}{2} - 5 = 25$ Solve for x.

a) 55
b) 60
c) 15
d) 35

QUIZIZZ

NAME : ______________________

CLASS : ______________________

DATE : ______________________

Solving Inequalities

NTI 19

10 Questions

1. Select all of the values that are solutions to $x \geq -8$.

a) -8.01

b) 8.01

c) 8

d) -8

2. When graphing do you use an open or closed circle for > ?

a) Open

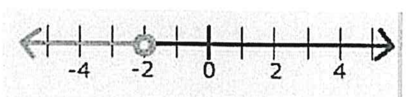
b) Closed

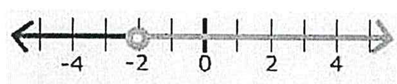
3. When graphing do you use an open or closed circle for $\leq$?

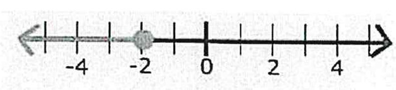
a) Open

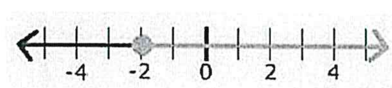
b) Closed

4. Which graph represents the inequality $2x + 3 < 7$

a) 

b) 

c) 

d) 

5. You spend at least 50 minutes practicing football each day. You spent 28 minutes practicing this morning. You want to practice twice as long in the afternoon. How long will you need to practice for?

a) $28 + 2x \leq 50$

b) $28 + 2x < 50$

c) $28 + 2x > 50$

d) $28 + 2x \geq 50$

6. $10 - 3x < 16$

a) $x < -2$

b) $x > -2$

c) $x < 2$

d) $x > 2$

7. $84 - 4x \leq 52$

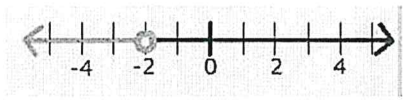
a) $x \geq 8$

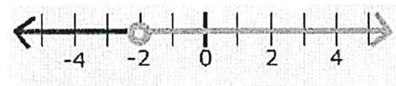
b) $x < 8$

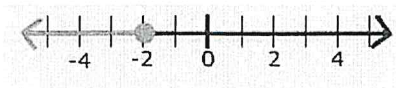
c) $x > 8$

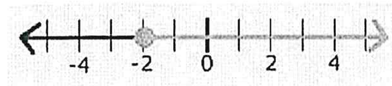
d) $x \leq 8$

8. Which graph represents x ≥ 2

a) 

b) 

c) 

d) 

9. Penelope is going to the carnival to ride the rides. It costs $20 to get into the carnival and ride tickets are $0.50 each. Write an equation that represents this scenario if she can spend no more than $35 in all.

a) $.50x + 20 \geq 35$

b) $.50x < 35$

c) $.50 + 20x \leq 35$

d) $.50x - 20 = 35$

10. A limo driver needs to make more than $450 in one day. He charges a rental fee of $375 and $0.85 per mile. What inequality represents the number of miles, x, he would have to drive to reach his goal?

a) $0.85x + 375 > 450$

b) $0.85x + 375 < 450$

c) $0.85 + 375x > 450$

d) $0.85 + 375x < 450$

Quizizz

Solving Inequalities

10 Questions

NAME : ____________________

CLASS : ____________________

DATE : ____________________

1. Select all of the values that are solutions to $x \geq -8$.

a) -8.01
b) 8.01
c) 8
d) -8

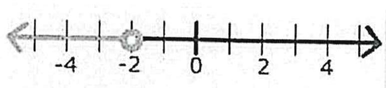
2. When graphing do you use an open or closed circle for > ?

a) Open
b) Closed

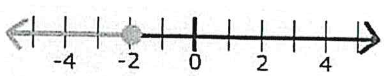
3. When graphing do you use an open or closed circle for $\leq$?

a) Open
b) Closed

4. Which graph represents the inequality $2x + 3 < 7$

a) 

b) 

c) 

d) 

5. You spend at least 50 minutes practicing football each day. You spent 28 minutes practicing this morning. You want to practice twice as long in the afternoon. How long will you need to practice for?

a) $28 + 2x \leq 50$
b) $28 + 2x < 50$
c) $28 + 2x > 50$
d) $28 + 2x \geq 50$

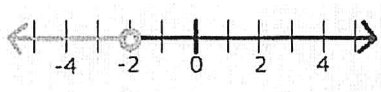
6. $10 - 3x < 16$

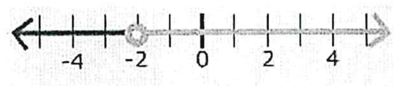
a) $x < -2$
b) $x > -2$
c) $x < 2$
d) $x > 2$

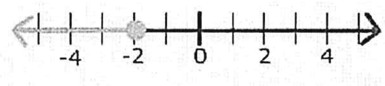
7. $84 - 4x \leq 52$

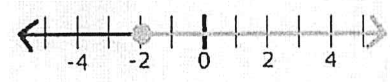
a) $x \geq 8$
b) $x < 8$
c) $x > 8$
d) $x \leq 8$

8. Which graph represents x ≥ 2

a) 

b) 

c) 

d) 

9. Penelope is going to the carnival to ride the rides. It costs $20 to get into the carnival and ride tickets are $0.50 each. Write an equation that represents this scenario if she can spend no more than $35 in all.

a) .50x + 20 ≥ 35

b) .50x < 35

c) .50 + 20x ≤ 35

d) .50x - 20 = 35

10. A limo driver needs to make more than $450 in one day. He charges a rental fee of $375 and $0.85 per mile. What inequality represents the number of miles, x, he would have to drive to reach his goal?

a) 0.85x + 375 > 450

b) 0.85x + 375 < 450

c) 0.85 + 375x > 450

d) 0.85 + 375x < 450

Calculus

Lesson 3: Separation of Variables

Topic 7.6: Finding General Solutions Using Separation of Variables

Equations where all x- terms can be collected with dx and all y -terms with dy are said to be separable. The solution procedure is called *separation of variables.* A differential equation will be in the form of $\frac{dy}{dx} = f(x) \cdot g(y)$. To solve, you must put your solution in the form of $y = h(x)$.

EX#1: Find the general solution for each of the following:

A. $\dfrac{dy}{dx} = \dfrac{2x}{y}$

B. $\dfrac{dy}{dx} = y^2$

C. $\dfrac{dy}{dx} = xy$

D. $4yy' - 3e^x = 0$

EX #2: Find the general solution of the differential equation $\dfrac{dy}{dx} = \dfrac{x^2 + 1}{3y^2}$

Topic 7.7: Finding Particular Solutions Using Initial Conditions and Separation of Variables

EX #3: Find the solution of the differential equation that satisfies the given condition.

$$xe^{-t}\frac{dx}{dt} = 1\,;\; x(0) = 1$$

EX #4: Find the solution of the differential equation that satisfies the given condition.

$$\frac{dy}{dx} = \frac{1 + x}{xy};\; y(1) = -4$$

EX #5: Find the solution of the differential equation that satisfies the given condition.

$$\sqrt{y}\,\frac{dy}{dx} + \sqrt{x} = 0\,;\; y(1) = 4$$

7.3 Separation of Variables Homework

Name ______________________

Date ______________ Period ________

Problems 1 – 6, Find the solutions to the differential equations, subject to the given initial conditions.

1. $6x + 5y\frac{dy}{dx} = 0$, $y(3) = 2$. Write your answer in standard form: $Ax^2 + By^2 = C$.

2. $\frac{dy}{dx} = \frac{y}{2\sqrt{x}}$, $y(4) = 1$

3. $\frac{dy}{dx} = (y + 8)$, $y(0) = 4$

4. $e^x - y\frac{dy}{dx} = 0$, $y(0) = 3$

13. The slope field for $y' = 0.25(2+y)(3-y)$ is shown at right.

A. Plot the following points on the slope field.

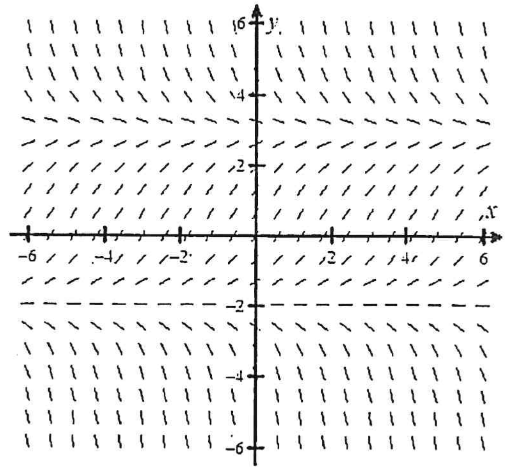
i.) $(0,0)$ ii.) $(2,0)$

iii.) $(0,-2)$ iv.) $(0,2)$

v.) $(0,-4)$ vi.) $(0,4)$

B. Plot solution curves through the points.

C. For which regions are all solution curves increasing? decreasing? or, have horizontal tangents?



14. Match the slope fields with the differential equations.

_______ **i.** $y' = 1 + y^2$

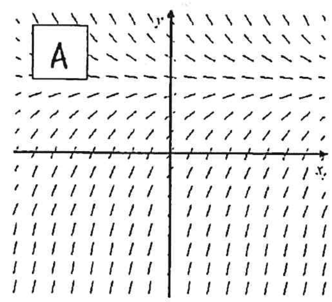
_______ **ii.** $y' = x$

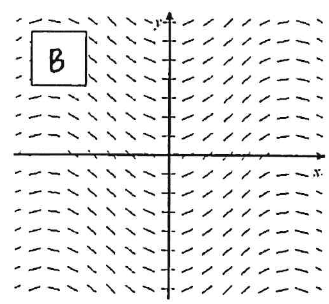
_______ **iii.** $y' = 2 - y$

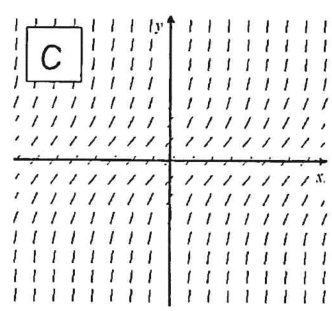
_______ **iv.** $y' = x - y$

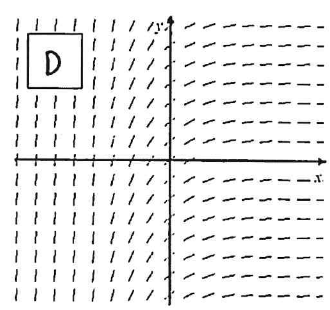
_______ **v.** $y' = \sin x$

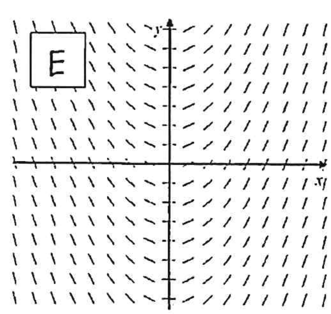
_______ **vi.** $y' = e^{-x}$

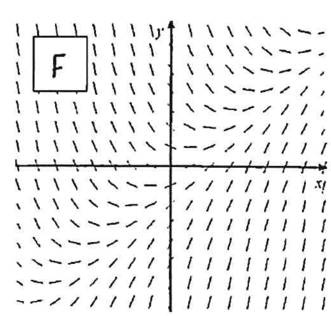












5. $dP + k(P - 65)dt = 0, P(0) = 130$

6. $2y\frac{dy}{dx} = x + \sin x, y(0) = 1$

Problems 7- 9, Find the general solution for each differential equation.

7. $(\sin\theta)\frac{dy}{d\theta} = \cos\theta$

8. $(x^2 - 1)\frac{dy}{dx} = \frac{2x}{\cos y}$

9. $\frac{dP}{dt} - aP = b$, Where a, b are nonzero constants.

Problems 10 – 11, Answer each of these questions without a calculator.

10. Write and solve the differential equation that models the following statement:
"The rate of change of Q with respect to x is proportional to $45 - x$."

11. The general solution of any differential equation is a family of functions. Determine the family of functions that corresponds to each of the following, given the choices:

I. parabolas **II.** circles **III.** hyperbolas

IV. lines **V.** exponential functions **VI.** logarithmic functions

A. $\dfrac{dy}{dx} = x$

B. $\dfrac{dy}{dx} = y$

C. $\dfrac{dy}{dx} = \dfrac{y}{x}$

D. $\dfrac{dy}{dx} = -\dfrac{x}{y}$

E. $\dfrac{dy}{dx} = \dfrac{k}{x}$

F. $\dfrac{dy}{dx} = \dfrac{x}{y}$

Lesson 4: Differential Equations - Growth and Decay

Topic 7.8: Exponential Models with Differential Equations

This lesson teaches how to use exponential functions to model growth and decay in applied problems. Consider the statement, "The rate of change of some quantity y is directly proportional to y."

$$\frac{dy}{dt} = ky$$

The Law of Exponential Change

If y is a differentiable function of t such that $y > 0$ and "***y* changes at a rate proportional to the amount present**" (that is, if $dy/dt = ky$), and for some constant k, then

$$y = Ce^{kt}$$

Where C is the initial value of y, and k is the constant of proportionality.

- ❑ When $k > 0$ Exponential growth occurs, and
- ❑ When $k < 0$ Exponential decay occurs.

EX #1: Prove that $\frac{dy}{dt} = ky$ supports the model $y = Ce^{kt}$

EX #2: The rate of change of y is proportional to y. When $t = 0, y = 3$, and when $t = 2, y = 6$. What is the value of y when $t = 3$?

EX #3: The number of bacteria in a culture is growing at a rate of $3000e^{\frac{2t}{5}}$ per unit of time, t. At $t = 0$ the number of bacteria present was 7,500. Find the number present at $t = 5$.

EX #4: A slow economy caused a company's annual revenues to drop from $530,000 in 2008 to $386,000 in 2010. If the revenue is following an exponential pattern of decline, what is the expected revenue for 2013?

EX #5: Suppose a population of insects increases according to the law of exponential growth. There were 130 insects after the third day of the experiment and 380 after the seventh day. Approximately how many insects were in the initial population?

Half-Life

If $A(t) = A_0e^{kt}$ with $k < 0$, then the half-life of A is

$$Half - life = \frac{\ln 2}{k}$$

EX #6: A zircon sample contains 4000 atoms of the radioactive element 235U. Given that 235U has a half-life of 700 million years, how long would it take to decay to 125 atoms?

EX #7: A cup of hot chocolate placed on a table cools at a rate if $\frac{dH}{dt} = -0.05(H - 75)$°F per minute, where H represents the temperature of the hot chocolate and t is time in minutes. If the coffee was at 160°F initially, what will its temperature be 10 minutes later?

7.4 Growth and Decay Homework

Name ______________________

Date ______________ Period ________

1. Write and solve the differential equation that models the following statement, "The rate of change of W with respect to x is proportional to $x + 18$."

2. In 10 years, the mass of a 200-gram sample of an element is reduced to 100 grams; this is the half-life of the element. Find k for this element.

3. Newtonium (Nt) is a highly radioactive element with a half-life of 625 years. How long would it take 62 grams to decay to 22 grams? (Round your answer to the nearest year.)

4. Bacteria from a lab culture grows in such a way that the instantaneous rate of change of the bacteria population is directly proportional to the number of bacteria present.

   A) Write a differential equation that expresses this relationship and solve the equation for the number of bacteria as a function of time.

   B) Initially there are 250 bacteria present. Three hours later, the culture has grown to 625. Find the bacteria population after one day.

   C) When will the bacteria culture grow to 100,000?

5. When processing raw sugar "inversion" causes the sugar's molecular structure to change. During inversion the rate of change of the amount of raw sugar, R, is proportional to the amount of raw sugar remaining. If 1000 kg of raw sugar reduces to 750 kg during the first 10 hours, how much raw sugar will remain at the end of a 24-hour period?

6. Let $P(t)$ represent the number of deer in a population at time t years, when $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to $700 - P(t)$, where the constant of proportionality is k.

   A) If $P(0) = 400$, find $P(t)$ in terms of t and k.

   B) If $P(2) = 600$, find k.

7. A bacteria colony is grown in a laboratory so that the population increases exponentially with time. At the end of 3 hours there are 5000 bacteria. At the end of the 5 hours there are 20,000 bacteria. How many bacteria were present initially?

8. A cup of coffee is placed in a 70°F room. The temperature of the coffee is 140 °F, initially and 102°F one hour later. Find the temperature of the coffee after 3 hours if the cooling rate is $\frac{dT}{dt} = k(T - 70)$

9. Warfarin™ sodium is a drug used for stroke victims as an anticoagulant. The half-life of the drug in the body is 38 hours. After the drug is stopped, the quantity remaining in a patient's body decreases at a rate proportional to the quantity remaining. How many days does it take for the drug level in the body to be reduced to 20% of the original level?

10. The half-life of morphine in the bloodstream of humans is 3 hours. If initially there is 0.4 mg of morphine in the bloodstream, find an equation for the amount in the bloodstream at any time. When does the amount drop below 0.01 mg?

11. Suppose a bacterial culture initially has 120 cells. After 2 hours, the population has increased to 500. Find an equation for the population at any time. What will the population be after 8 hours?

AP Calculus
Differential Equations Test #7-A

Name ______________________

Calculator is NOT ALLOWED on this portion of the test!

1. Solve the equation. $f'(x) = 6f(x)$ and $f(2) = 9$

A. $f(x) = 6e^{9x+2}$

B. $f(x) = 9e^{2x+6}$

C. $f(x) = 9e^{6x-12}$

D. $f(x) = e^{6x+9}$

---

2. If $\frac{dy}{dt} = y\cos t$ and $y = 3$ when $t = 0$, then $y =$

A. $e^{\sin t} + 3$

B. $3e^{\sin t}$

C. $sin\, t + 3$

D. $\sin t + 3e^{t}$

---

3. A bacteria culture starts with 200 bacteria and triples in size every half hour. After 2 hours, how many bacteria are there?

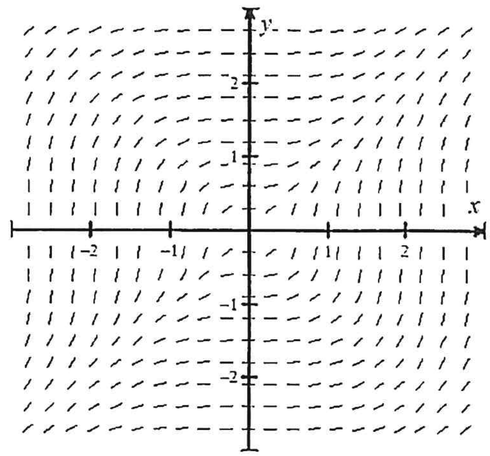
A. 16,200

B. 17,800

C. 19,300

D. 23,500

4. Shown at right is the slope field for which differential equation?



A. $\frac{dy}{dx} = \frac{x^3}{y^2}$

B. $\frac{dy}{dx} = \frac{x^3}{y}$

C. $\frac{dy}{dx} = \frac{x^2}{y^2}$

D. $\frac{dy}{dx} = \frac{x^2}{y}$

---

5. The rate of growth of a bacteria population is proportional to the number of bacteria present. The initial population quadrupled during the first 2 days. By what factor will the population have increased during the first 3 days?

A. 4 B. 5

C. 6 D. 8

---

6. The amount of pain reliever in Tony's bloodstream decreases at a rate that proportional at any time, to the amount of pain medication in the bloodstream at that time. Tony takes 180 milligrams of the pain reliever initially. The amount of medication is halved every 3 hours. How many milligrams of the medication are in Tony's bloodstream after 8 hours?

A. 28 mg B. 68 mg

C. 116 mg D. 143 mg

7. Which of the following differential equations has $x^2 - 4y^2 = 8$ as a solution?

A. $\frac{dy}{dx} = 4xy$

B. $\frac{dy}{dx} = \frac{4x}{y}$

C. $y\frac{dy}{dx} = \frac{x}{4}$

D. $\frac{dy}{dx} = -\frac{x}{4y}$

---

8. If $\frac{dy}{dx} = \frac{x}{y}$ and $y(2) = 5$, then

A. $x^2 - y^2 = -21$

B. $x^2 + y^2 = 21$

C. $y^2 - x^2 = 11.5$

D. $x^2 - y^2 = 21$

---

9. Contaminated water is being pumped continuously into a tank at a rate that is inversely proportional to the amount of water in the tank; that is, $\frac{dy}{dt} = \frac{k}{y}$, where y is the number of gallons of water in the tank after t minutes $(t \geq 0)$. Initially, there were 5 gallons of water in the tank, and after 3 minutes there were 7 gallons. How many gallons of water were in the tank at $t = 18$ minutes?

A. 17

B. $\sqrt{97}$

C. $\sqrt{61}$

D. 13

Questions 10 and 11 refer to the following information:

Consider the differential equation $\frac{dy}{dx} = \frac{\sin x}{y}$, for which the solution is $y = f(x)$. Let $f(0) = 1$

10. Which of the following statements about the graph of $f(x)$ are true?

I. $f(x)$ has a vertical tangent when $y = 0$

II. $f(x)$ has a horizontal tangent when $x = 0$

III. The slope of $f(x)$ at the point (0,1) is 1.

A. I only

B. II only

C. I and II only

D. I, II, and III

---

11. The particular solution is

A. $f(x) = \sqrt{2\cos x + 3}$

B. $f(x) = \sqrt{3 - 2\cos x}$

C. $f(x) = -e^{\cos x}$

D. $f(x) = 3e^{-2\cos x}$

---

12. If a substance decomposes at a rate proportional to the amount of the substance present, and the amount decreases from 60 ounces to 15 ounces in 2 hours, find the constant of proportionality.

A. $-\ln 2$

B. $-\frac{1}{2}$

C. $\ln\frac{1}{4}$

D. $-\frac{1}{4}$

13. A radioactive element decomposes at a rate proportional to the amount present. The amount remaining after t years, A_F, can be represented in terms of the initial amount A_0 where k is the negative constant of proportionality is

A. $A_F = A_0\, kt$

B. $A_F = A_0\, e^{kt}$

C. $A_F = e^{kt}$

D. $A_F = A_0 + \frac{1}{2}kt^2$

---

14. If $e^y \frac{dy}{dx} = 3x^2$ and $y(1) = 2$, then the particular solution $y(x)$ is

A. $y = \ln x^3 + 2$

B. $y = 2e^{x^3 - 1}$

C. $y = \ln\left(x^3 + e^2 - 1\right)$

D. $y = x^3 + e^2 - 1$

Free Response

Calculator is REQUIRED on this portion of the test!

The radiation $R(t)$ in a substance decreases at a rate that is proportional to the amount present; that is, $\frac{dR}{dt} = kR$, where k is the constant of proportionality and t is the time measured in years. The initial amount of radiation is 7600 rads. After three years, the radiation has declined to 500 rads. (Note: One rad $= 0.01\frac{J}{kg}$ is a unit used to measure absorbed radiation doses).

A) Express R as a function of time.

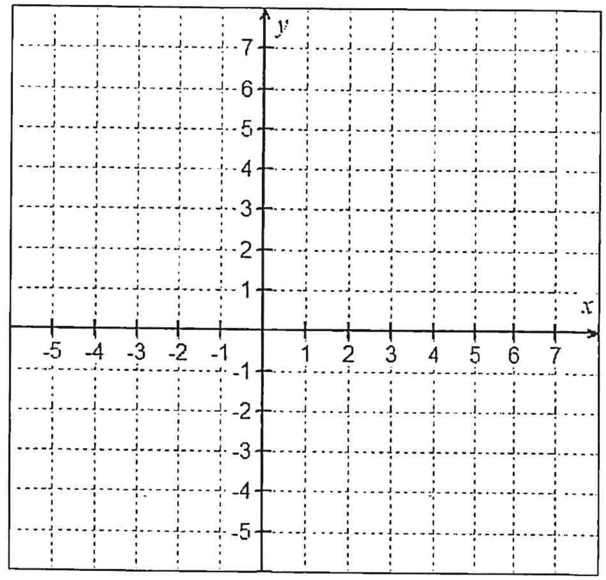
B) When will the radiation drop below 20 rads?

C) Find the half-life of this substance.

6.1 Area Between Two Curves

In this unit we will explore more applications of integration by investigating the area between two curves and three-dimensional volumes.

EX #1: On the grid at right,



A. Graph the function $f(x) = -\frac{1}{2}(x-2)^2 + 6$

B. Find the value of $\int_0^5 f(x)\,dx$.

C. Shade the region blue.

D. Graph the function $g(x) = 4 - \frac{1}{2}x$

E. Find the value of $\int_0^5 g(x)\,dx$.

F. Shade the region green.

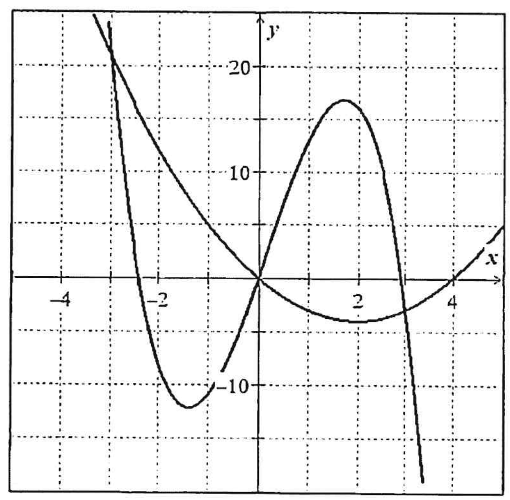
G. How would you find the area of the region that lies between the two graphs $f(x)$ and $g(x)$?

This leads to the general approach we will use for finding the area between two curves.

EX#2: Find the area between the two curves algebraically. Show your work.

$$A = \int_0^5 [f(x) - g(x)]\,dx$$

EX #3: Find the Area of the Region between the graphs of $f(x) = 14x + x^2 - 2x^3$ and $g(x) = x^2 - 4x$



A. Find the intersection of the two curves.

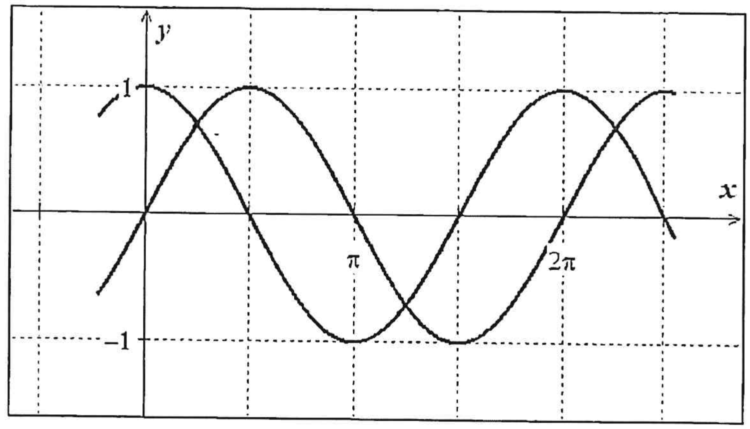
B. Write the integrals and calculate the area.

EX #4: Find the area of <u>one</u> of the regions bounded by the sine and cosine curves.

A. Sketch note $f(x) \geq g(x)$

$f(x) =$

$g(x) =$

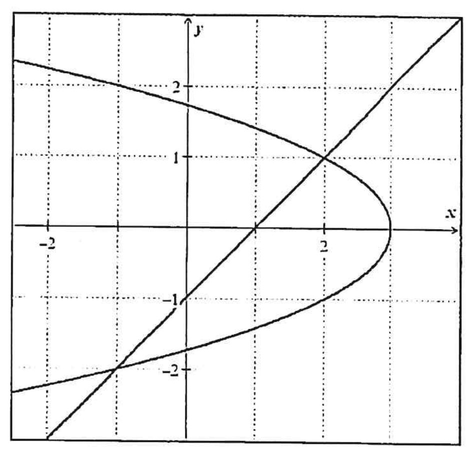


B. Find bounds $[a, b]$ where $\sin x = \cos x$

C. Integrate

EX #5: Horizontal Rectangles (*Y*-Axis Approach)
Find the area of region bounded by graphs of $x = 3 - y^2$ and $x = y + 1$

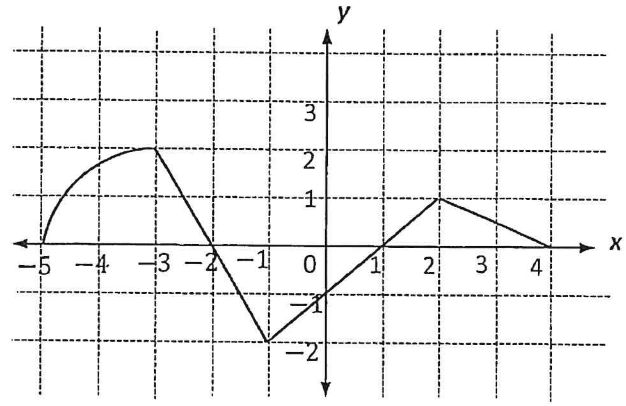
NOTE: w/r/t/ x → need two integrals for $[-1, 2]$ *and* $[2, 3]$
w/r/t/ y → need only one integral for $[-2, 1]$



EX #6: Area as an Accumulation Process-Revisited

The graph of the function g consists of a quarter-circle and three line segments, shown below. Let f be the function defined by $f(x) = \int_{-1}^{x} g(t)dt$.

A. Find $f(-5)$
B. Find all the values of x on the open interval $(-5, 4)$ where f is decreasing. Justify your answer.
C. Write an equation for the line tangent to the graph of f at $x = -1$.
D. Find the minimum value of f on the closed interval $[-5, 4]$. Justify your answer.



EX #7: The density of cars on 10 miles of the highway approaching Disney World is approximated by $f(x) = 180[4 - \ln(2x + 3)]$, where x is the distance in miles from the Magic Kingdom entrance. Find the total number of cars on this 10-mile stretch.

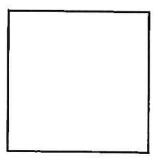
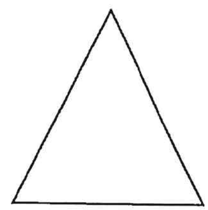
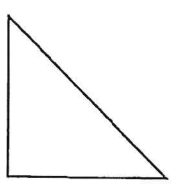
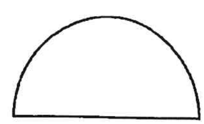
6.2 Volumes by Cross Sections

In this lesson we will use calculus to find volumes of three-dimensional shapes where the cross sections are geometric shapes like squares, triangles, semi-circles, rectangles and such. Finding the formula for the area of the cross section is the tricky part. But, once you figure that out, then just integrate like you've been doing. In general, if the cross-sections are perpendicular to the x-axis, solve the equations for y. If the cross-sections are perpendicular to the y-axis, solve the equations for x.

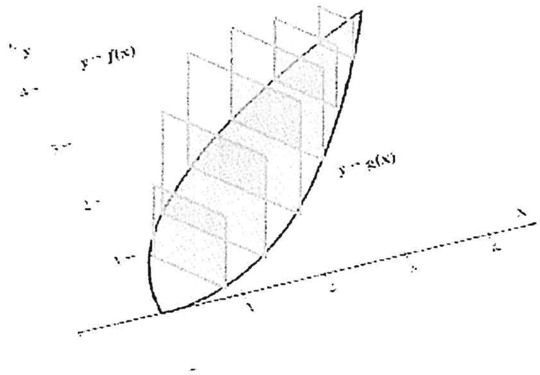
For cross sections of area $A(x)$ taken perpendicular to an axis:

CROSS SECTIONS TAKEN PERPENDICULAR TO X-AXIS	$V = \int_a^b A(x)\,dx$
CROSS SECTIONS TAKEN PERPENDICULAR TO Y-AXIS	$V = \int_c^d A(y)\,dy$

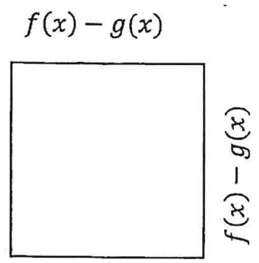
Do you recall the area formulas from Geometry?

SQUARES 	EQUILATERAL TRIANGLES 
ISOSCELES RIGHT TRIANGLES 	SEMI CIRCLES 

Square Cross Sections

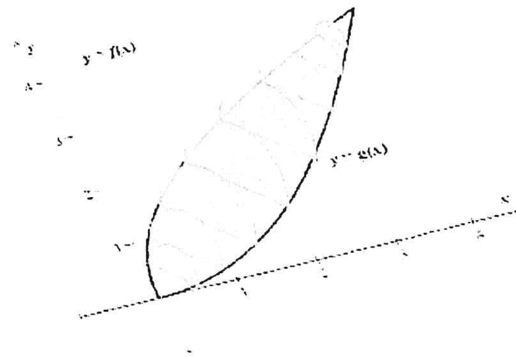


First, write the area of one square in terms of the given functions.

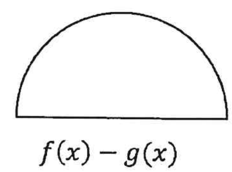


By adding an infinite number of squares, the total of the cross-sectional areas would give a volume of the solid created. Write this volume on the interval $[a, b]$ as an integral.

Semi-Circle Cross Sections

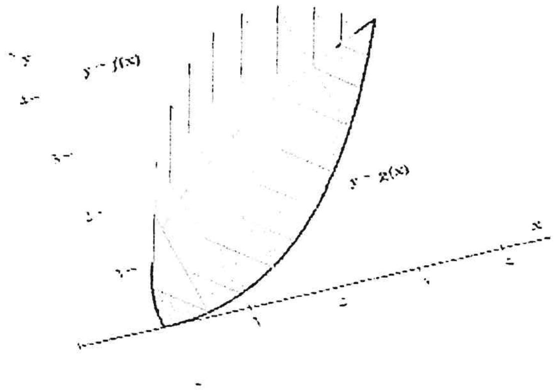


Semi-circles are a little tricky. In this case you need to find the radius in terms of the functions. Then, write the area for one semi-circular cross section.

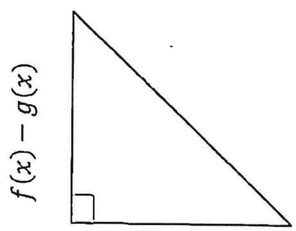


Now, write the volume of this solid generated by slicing the region between the curves of $f(x)$ and $g(x)$ on the interval $[a, b]$.

Isosceles Right Triangle Cross Sections



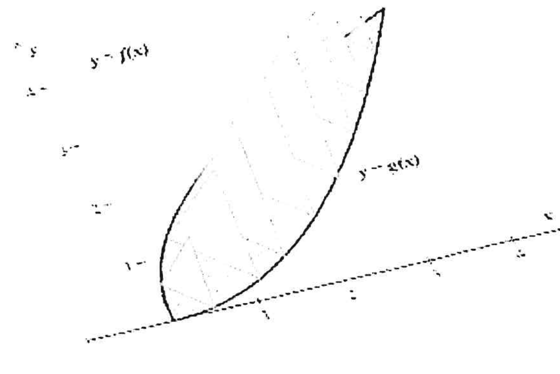
Write the area of the triangle in terms of $f(x)$ and $g(x)$.



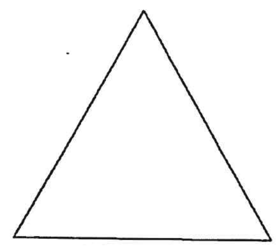
$f(x) - g(x)$

Write the volume of this solid generated as an integral in terms of $f(x)$ and $g(x)$ on the interval $[a, b]$.

Equilateral Triangle Cross Sections



Use properties of 30-60-90 triangles to write the area in terms of $f(x)$ and $g(x)$.



$f(x) - g(x)$

Use calculus to write the volume of the solid generated by summing the areas of an infinite number of triangles on the interval $[a, b]$.

EX #1: Find the volume of the solid whose base is bounded by the function $y = \sqrt{x}, x = 4$ and the x-axis. Each cross section is a square, taken perpendicular to the x-axis.

EX #2: Find the volume of the solid whose base is a region bounded by the circle $x^2 + y^2 = 9$ with the cross sections taken perpendicular to the x-axis are isosceles right triangles with the hypotenuse lying on the base of the circle.

EX #3: Let R be the region bounded by the graphs of $x = y^2$ and $x = 9$. Find the volume of the solid that has R as its base if every cross section by a plane perpendicular to the x-axis is a semi-circle.